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Optimal Robust LPV Control Design for Novel Covid-19 Disease

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Abstract: These days almost all countries around the world are struggling with coronavirus outbreak. If the governments and public health care systems don't take any action against this outbreak, it would have severe effects on human life, now and in the future. By doing so, there are several intervention strategies that could be implemented and as the result, the societies become more secure from the casualties of this virus. In this paper, we used a mathematical model of coronavirus epidemic transmission and by use of some LMIs, a robust LPV controller is designed which helps us to choose and use the intervention methods, effectively. By use of the proposed robust controller, the robustness and stability of the model against a wide range of uncertainties are approved. The final objective of this control design is to minimize the number of exposed and infected individuals in the compartmental model. In the end, it can be seen that the control strategies which are preventive action, good medical care, and sterilization of the environment, can highly reduce the negative effects of the coronavirus.

Keywords: Covid-19; mathematical model; robust controller; intervention strategies; LMI; uncertainties.

طراحی کنترلر بهینه مقاوم LPV برای بیماری جدید کوید-۱۹

رضا نجارزاده، مریم دهقانی، محمدحسن آسمانی، روزبه ابولپور

چکیده: این روزها تقریباً همه کشورهای جهان با شیوع ویروس کرونا دست و پنجه نرم می کنند. اگر دولت ها و سیستم های بهداشت عمومی اقدامی در برابر شیوع این بیماری انجام ندهند ، اکنون و در آینده تأثیرات شدیدی بر زندگی انسان خواهد داشت ، ازین سو ، چندین استراتژی مداخله ای وجود دارد که می تواند اجرا شود و در نتیجه ، جوامع از تلفات این ویروس ایمن تر می شوند. در این مقاله ، ما از یک مدل ریاضی انتقال اپیدمی ویروس کرونا استفاده کرده ایم و با استفاده از تعدادی LMI ، یک کنترل کننده LPV مقاوم طراحی شده است که به ما در انتخاب و استفاده از روش های مداخله به طور موثر کمک می کند. مدل ویروس کرونا حاوی عدم قطعیت ها در برخی متغیر های حالات و مقادیر پارامترها است. بنابراین ، با استفاده از کنترل کننده مقاوم پیشنهادی ، مقاوم بودن و پایداری مدل در برابر طیف گسترده ای از عدم قطعیت ها تأیید می شود. هدف نهایی طراحی این کنترلر ، به حداقل رساندن تعداد افراد در معرض و آلوده در مدل بخش بخش است. در پایان ، می توان دریافت که استراتژی های کنترلی شامل اقدامات پیشگیرانه ، مراقبت های پزشکی خوب و ضدعفونی سازی محیط می توانند اثرات منفی ویروس کرونا را تا حد زیادی کاهش دهند.

کلمات کلیدی: کوید-۱۹، مدل ریاضی، کنترلر مقاوم، استراتژی های مداخله ای، نامساوی خطی ماتریسی، عدم قطعیت

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1.Introduction

The coronavirus outbreak started in late 2019 and early 2020, originated in the Hubei province of China and Wuhan City. The coronavirus which caused the 2019 and 2020 outbreak is from the family of SARS-associated coronavirus. This virus caused three outbreaks until this day. The first outbreak was named SARS (severe acute respiratory syndrome) caused by SARS-CoV-1, was first discovered in China in February 2003 [1]. During this epidemic, there were 8,422 confirmed infected cases and the fatality rate of this virus was about 11% [2]. The second outbreak was first emerged in Saudi Arabia in September 2012 and caused by Middle East Respiratory Syndrome Coronavirus (MERS-CoV) [3]. For this epidemic, there were 2,500 confirmed infected cases and the fatality rate of the disease was about 35% [4]. The third and also the last outbreak of this virus is the ongoing epidemic we are facing these days. This disease is caused by the SARS-CoV-2 virus and by Feb 2021 there are more than 110 million cases and about 2.4 million people who have died from this disease [5]. The fatality rate of this virus is estimated about 2% [6]. To describe the transmissibility of this virus, we introduce a factor which is R_0. This number shows every single person who has the virus, can infect how many other individuals. First, WHO (world health organization) estimated the R_0 between 1.4 and 2.5 [7]. But later, some studies showed a value of 3.6 and 4 and the others showed a value of 2.24 and 3.58 for R_0 [8]. For comparison, the R_0 of two previous viruses were about two for SARS and less than one for MERS. As you can see, the last type of virus spread much faster than the two others, but with a much less fatality rate.

Fortunately, some vaccines are discovered by this day, but because the process of vaccination and producing vaccines are time-consuming, the intervention strategies should be continued. Here, three intervention methods are considered. The first one is preventive actions which make the contact between people less and less like quarantine, social distancing, and isolation of infected individuals. The second one is good medical care like using some auxiliary medicines which help infected individuals to recover faster and better from the disease. The third one is sterilizing and disinfection measures to clean our body and our environment from the density of the virus. These intervention strategies could prevent a sudden increment in the number of infected individuals and also increase the number of recovered people. For this purpose, a mathematical model is needed to describe the transmission of the coronavirus. In fact, without having a proper model, we cannot estimate the behavior of the system well, so our simulation may contain unrealistic results.

Mathematical models use the key factors of disease transmissions such as getting reinfected, infected individuals with or without symptoms, transmission rate between different people, contact rate, etc. In this case, like all other pandemic models, most of the proposed models are based on the SEIR model. In such models, the population of a society based on their health condition are divided into different compartments, and because of this, these kinds of models are called compartmental models. Models could be in a continuous-time form or a discrete-time form. In this paper, we used a continuous model which is introduced in the next section, but for more details about discrete-time models, readers are referred to [9-12]. There were several studies about modeling the coronavirus transmission from early 2020 and now on [13-16]. Jana et al. in [17] used an SEIR epidemic model to study the role of transportation between two cities in the transmission of the disease. They concluded that transportation may cause a big change in the dynamics of the model, and it increases the probability of the virus being transferred between the people of those two cities. This factor may create impulsive changes in virus spread and thus impulsive controller is needed to handle this effect [18]. Leung et al. used the first data extracted from the Hubei province of China which was the center of the virus spread to describe the virus transmission behavior [13]. The city Wuhan and the other cities next to it have been locked down since Jan 23 and it was the first intervention action that was used, there. Then schools have been closed, and the other non-essential jobs have restricted their activities. Face masks and social distancing became obligatory to prevent disease spread. The effectiveness of these intervention methods is also studied by considering the number of patients and active infected and recovered individuals [13]. Kucharski et al. [14] also studied the modeling of the virus spread. They mentioned that the first thing to notice in pandemic events is the transmission dynamics of the disease. Also, studying the changes which are likely to happen during the time for the transmission dynamics, help us to find out and estimate the future behavior of the disease spread and also the effectivity of control measures that are taken into account by that stage of the pandemic. Sameni [12] also used SIR-based model for coronavirus spread and showed that how actions like quarantine, isolation, lockdowns, medical caution, etc. can affect the parameters of the models like contact rate, mortality rate and the number of infected individuals, respectively.

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Several types of controllers are designed for the model of the coronavirus, lately [6, 19-23]. Lemecha et al. [6] proposed an optimal control approach on the coronavirus model. The objective of this controller is to minimize the number of infected and exposed individuals with the respect to the cost of control implementations. They used Pontryagin's Maximum Principle to find and formulate this controller design. They also obtained a mathematical equation for basic reproduction number (R_0) with regards to controller inputs. Then, the sensitivity of this factor is studied, considering the parameters of the model. The simulation results show that the proposed control intervention strategies reach the objective of the problem with optimum cost. Péni et al. [22] designed a model predictive control for the constrained compartmental discrete-time model of the disease. This discrete model could properly manage the complexities and the relation between parameters and states and also different intervention stages. In that work, five different control problems with objectives and costs are studied including an output feedback schematic by use of numerical simulations. A state observer is also designed to estimate the parameters uncertainties and also non-measured parameters of the model. The results show that fast, on time and continuous interventions could practically prevent the profuse number of infected individuals. Rohith et al. [21] model the dynamics of the disease with an SEIR model considering a nonlinear incident rate as a control strategy applied by the government. A bifurcation analysis is also proposed to find out how different basic reproduction numbers (R_0) can change coronavirus transmission procedure. Then, a robust closed loop sliding mode control is designed for the model and as the result, they showed that by use of this controller, the value of R_0 can be lowered to one from its initial value (2.5).

One of the challenges we are facing in using a model for a pandemic transmission, is uncertainties in parameters. In this paper, we used a continuous-time SEIRV model considering the parameters uncertainties. To solve this problem, a robust LPV controller is designed using a feedback control configuration which can be robust against a wide range of uncertainties of parameters. Then, a cost function is considered and the optimal control problem is solved and simulated which shows that our controller could behave properly.

The paper is organized in the following order: In section II, the mathematical SEIRV model of the system is presented. In section III, a polytopic LPV model of the proposed model is formulated and a feedback controller is designed and utilized for the system. In section IV, our simulation results are shown and we compare the obtained results in different cases. In section V, the conclusion and discussion of our work are given.

2. Mathematical model of coronavirus transmission

For this study, we used a SEIRV model of coronavirus transmission presented in [6]. In this model, there are three control inputs u_1, u_2, u_3 . The first control input represents preventive actions like quarantine isolation and lockdowns which lower the contact rate between different groups of people in a society. The second one is good and serious medical care that help infected individuals to recover from the disease as fast as possible. The last control input represents the sterilization and disinfection measures like washing hands, using antiseptic sprays, etc. which reduces the density of coronavirus in our environment and on surfaces, and our body. The system is described with the following equations:

$$\begin{cases} \frac{dS}{dt} = \Lambda - \left(1 - u_1(t)\right) \\ \begin{pmatrix} \beta_E(E)SE + \beta_I(I)SI \\ +\beta_V(V)SV \end{pmatrix} - \mu S \\ \frac{dE}{dt} = \left(1 - u_1(t)\right) \\ \begin{pmatrix} \beta_E(E)SE + \beta_I(I)SI + \\ \beta_V(V)SV \end{pmatrix} - (\alpha + \mu)E \\ \frac{dI}{dt} = \alpha E - \left(\omega + \gamma + \mu + u_2(t)\right)I \\ \frac{dR}{dt} = (u_2(t) + \gamma)I - \mu R \\ \frac{dV}{dt} = \xi_1 E + \xi_2 I - \left(\sigma + u_3(t)\right)V \end{cases} \tag{1}$$

Where five states are: Susceptible *S(t)* or people who don't get the infection yet, but they are prone to disease, Exposed *E(t)* or people who are infected, but they are not infectious, Infected *I(t)* or people who are infected by the disease and show some symptoms, Recovered *R(t)* or people who are recovered and healed from the disease and *V(t)* is the density of coronavirus in our environment and surfaces. It can be noted that all the states should be nonnegative. Three following nonlinear and non-increasing functions are considered for describing the transmission rate between exposed and susceptible individuals ($\beta_E(E)$), infected and susceptible individuals ($\beta_I(I)$) and virus transmission rate between humans and the environment ($\beta_V(V)$) [6].

$$\beta_E(E) = \frac{\beta_{E0}}{1+cE}, \beta_I(I) = \frac{\beta_{I0}}{1+cI}, \beta_V(V) = \frac{\beta_{V0}}{1+cV} \tag{2}$$

The above three variables are non-negative. These equations show that if the number of *E*, *I*, and *V* increases, a stronger and higher value of control inputs are needed in (1) to attenuate the effects of those increments. The parameters of the model are described in the "Table I".

The overall population of the assumed society is *N* and there is an algebraic relation between the states of the model as *S=N-(E+I+R)* [6]. Also, without having vaccination as a control input, these three control inputs can't create herd immunity for the population of society. So, everyone who gets infected and then recovered can get the infection again, in the future. Having this situation in model, the state S or susceptible individuals remains a positive constant value which equals to its initial condition. For more simplification, we assume that $1-u_1(t)=u_{1n}(t)$. Then, we have the simplified model as:

$$\begin{cases} \frac{dE}{dt}=u_{1n}(t) \\ \begin{pmatrix}\beta_E(E)SE+\beta_I(I)SI+\\ \beta_V(V)SV\end{pmatrix}-(\alpha+\mu)E \\ \frac{dI}{dt}=\alpha E-(\omega+\gamma+\mu+u_2(t))I \\ \frac{dR}{dt}=(u_2(t)+\gamma)I-\mu R \\ \frac{dV}{dt}=\xi_1 E+\xi_2 I-(\sigma+u_3(t))V \end{cases} \tag{3}$$

Table I. PARAMETERS OF CORONAVIRUS TRANSMISSION MODEL [6]

Parameter	Definition	Value
Λ	Influx rate	271.23 per day
β_{E0}	Transmission constant between S and E	3.11×10^{-8}/(person)/(day)
β_{I0}	Transmission constant between S and I	0.62×10^{-8}/(person)/(day)
β_{V0}	Transmission constant between S and V	1.03×10^{-8}
μ	Natural death rate	3.01×10^{-5} per day
c	Transmission adjustment coefficient	1.01×10^{-4}
ξ_1	Virus shedding rate by exposed people	2.30 /(ml)/(person)/(day)
ξ_2	Virus shedding rate by infected people	0/(ml)/(person)/(day)
α	Inverse incubation period	1/7 days
ω	Disease-induced death rate	0.01 per day
γ	Recovery rate	1/15 per day
σ	Removal rate of virus	1 per day

Ignoring the control inputs, the basic reproduction number (R_0) for the system (1) is obtained as [24]:

$$R_0=\left[\frac{\beta_E(0)}{\alpha+\mu}+\frac{\beta_I(0)\alpha}{(\alpha+\mu)\psi}+\frac{\beta_V(0)(\xi_1+\xi_1\psi+\xi_2\alpha)}{(\alpha+\mu)\psi\sigma}\right]\frac{\Lambda}{\mu} \tag{4}$$

Where $\psi=(\omega+\gamma+\mu)$. This number shows how contagious is a pandemic and if the value of it is one or less than one, the disease dies out, gradually. In contrast, if the value of R_0 is higher than one (like coronavirus which its R_0 is about 2.5 [5]) serious intervention strategies are required to prevent the virus spread more and more and to reduce the mortality rate to the minimum possible amount. The following expression describes obtaining R_0 with respect to control inputs [6]:

$$R_0=\left[\frac{\beta_E(0)}{\alpha+\mu}\frac{\beta_I(0)\alpha}{(\alpha+\mu)(\psi+u_2)}\frac{\beta_V(0)(\xi_1+\xi_1\psi+\xi_2\alpha)}{(\alpha+\mu)(\psi+u_2)(\sigma+u_3)}\right]\frac{u_{1n}(t)\Lambda}{\mu} \tag{5}$$

As it can be observed from (4), R_0 and control inputs have inverse relation, meaning that by increasing the value of control inputs, the value of R_0 decreases and vice versa. Here, we aim to design the control input such that R_0 becomes less than one.

3.Polytopic LPV model

A.LPV formulation

In this section, we present the LPV form [25, 26] of (3) considering uncertainties in some parameters. As you can see in (2), the transmission rate has a nonlinear expression. Because we don't know the exact rate of transmission, we consider some uncertainties for these parameters. The source of varying uncertainties is three states E, I, and V which vary in an interval. The equilibrium point of the system is obtained as:

$$eq.\ point=(E_0,I_0,R_0,V_0)=(0,0,0,c) \tag{6}$$

Where c is a positive steady state constant. We used the above equilibrium point and then the nonlinear system (3) is linearized by the use of the Jacobian method as follows:

$$\begin{cases}\dot{x}=A(r)x+B(r)u\\ y=C(r)x+D(r)u\end{cases} \tag{7}$$

Where $x\in\mathbb{R}^n$ are states, u are control inputs and y is the output of the system, respectively and r shows the varying parameters of state uncertainty. In this system $D(r)=0$ and also the output is not dependent on uncertainties, so $C(r)=C$. Since the states appear in the matrix B, we considered them as varying parameters in the LPV model. The entry of second row and second column of A matrix consist of several parameters, therefore it might be uncertain with more probability than the other entries of matrix and for instance we considered ω as the uncertain parameter in the prementioned entry because it seems to be harder to calculate the exact value of this parameter. So, in this model there will be four uncertainties as r_1 for the state E, r_2 for the state I and r_3 for the state V and r_4 for ω which is disease-induced death rate. Therefore, A, B and C matrices are quantized as follows:

$$A=\begin{bmatrix}-(\alpha+\mu) & 0 & 0 & 0\\ \alpha & -(r_4+\gamma+\mu) & 0 & 0\\ 0 & \gamma & -\mu & 0\\ \xi_1 & \xi_2 & 0 & -\sigma\end{bmatrix},$$

$$B=\begin{bmatrix}-\beta_{E0}S_0r_1+\beta_{I0}S_0r_2+\beta_{V0}S_0r_3 & 0 & 0\\ 0 & -r_2 & 0\\ 0 & r_2 & 0\\ 0 & 0 & -r_3\end{bmatrix},$$

$$C=[1\ 1\ 0\ 0] \tag{8}$$

Where r_1, r_2, r_3, r_4 are four uncertainties for states *E*, *I*, *V* and the parameter ω, respectively.

It can be noticed that in (8), *A* matrix has an entry with some uncertainties and *B* matrix is made of three entries that include some uncertainties. So, the polytopic model [27-29] of this system is like a multi-dimensional space which has 16 vertices. It means that there are 8 different *B* matrices and two different *A* matrices as:

$$B = B_1\delta_1 + B_2\delta_2 + \cdots + B_8\delta_8, A = A_1\delta_1 A_2\delta_2 \quad (9)$$

Where δ_i represent uncertainties. So, both *B* and *A* matrices have a unique value for each of the different values of uncertain parameters. Therefore, we should study the polytopic model in a space with 16 vertices, and the stability of the whole system is approved if and only if every vertex of space is stable. A sector nonlinearity approach is used to get a global sector in which the model $\dot{x} = f(x(t))$ could fit. for each uncertain parameter, a nonlinear function $\phi(x): \mathbb{R} \to \mathbb{R}$ is fitted in a sector (b_1, b_2) for all $x \in \mathbb{R}$, $y = f(x)$ stand between $b_1 x$ and $b_2 x$ [30]. This model approves the stability of f under the control system law.

By using the above definition and "Fig. 1" we have:

$$b_1 x \le f(x) \le b_2 x \to f(x) = d_1 b_1 x + d_2 b_2 x \ , \ d_1 + d_2 = 1 \ , 0 \le d_1, d_2 \le 1 \quad (10)$$

Where $d_1 = \frac{b_2 x - f(x)}{(b_2 - b_1)x}$, $d_2 = \frac{f(x) - b_1 x}{(b_2 - b_1)x}$. Then, the polytopic model can be obtained as:

$$\dot{x} = f\big(x(t)\big) = \sum_{i=1}^{2} d_i (A_i x) \quad (11)$$

Then, we can rewrite (10) as:

$$b_1 \le \frac{f(x)}{x} = g(x) \le b_2 \to g(x) = d_1 b_1 + d_2 b_2 \ , \ d_1 + d_2 = 1 \ , 0 \le d_1, d_2 \le 1 \quad (12)$$

Where $d_1 = \frac{b_2 - f(x)}{(b_2 - b_1)}$, $d_2 = \frac{f(x) - b_1}{(b_2 - b_1)}$. So, the final polytopic model for a single uncertain parameter is formulated as follows:

$$\dot{x} = x(t) g\big(x(t)\big) = \sum_{i=1}^{2} d_i (A_i x) \quad (13)$$

As we said before, four uncertain parameters including states *E*, *I* and *V* and the parameter ω were considered in the proposed model. First, we have to convert the uncertain parameters into affine form like the above procedure as:

$$\begin{cases} E = d_1 \underline{E} + d_2 \overline{E} \ , \ d_1 + d_2 = 1 \ , \\ 0 \le d_1, d_2 \le 1 \ , \ d_1 = \frac{\overline{E} - E}{\overline{E} - \underline{E}} \ , \ d_2 = \frac{E - \underline{E}}{\overline{E} - \underline{E}} \\ I = d_3 \underline{I} + d_4 \overline{I} \ , \ d_3 + d_4 = 1 \ , \\ 0 \le d_3, d_4 \le 1 \ , \ d_3 = \frac{\overline{I} - I}{\overline{I} - \underline{I}} \ , \ d_4 = \frac{I - \underline{I}}{\overline{I} - \underline{I}} \\ V = d_5 \underline{V} + d_6 \overline{V} \ , \ d_5 + d_6 = 1 \ , \\ 0 \le d_5, d_6 \le 1 \ , \ d_5 = \frac{\overline{V} - V}{\overline{V} - \underline{V}} \ , \ d_6 = \frac{V - \underline{V}}{\overline{V} - \underline{V}} \\ \omega = d_7 \underline{\omega} + d_8 \overline{\omega} \ , \ d_7 + d_8 = 1 \ , \\ 0 \le d_7, d_8 \le 1 \ , \ d_7 = \frac{\overline{\omega} - \omega}{\overline{\omega} - \underline{\omega}} \ , \ d_8 = \frac{\omega - \underline{\omega}}{\overline{\omega} - \underline{\omega}} \end{cases}$$

$$\delta_1 = d_1 * d_3 * d_5 * d_7 \ , \ \delta_2 = d_1 * d_3 * d_5 * d_8 \ , \ldots, \delta_{16} = d_2 * d_4 * d_6 * d_8 \quad (14)$$

Where δ_i are the uncertainties of the LPV model. The polytopic space was supposed to have 16 vertices.

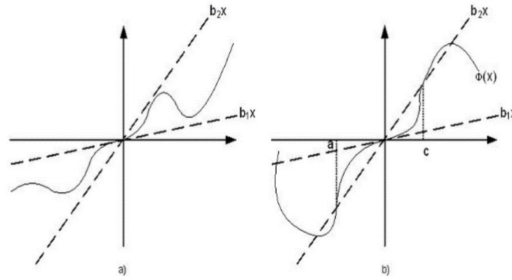


Figure 1.a) Global sector nonlinearity, b) Local sector nonlinearity [30]

So, by use of the above definitions, the polytopic LPV model of the system is obtained as:

$$\dot{x} = \sum_{i=1}^{16} \delta_i (A_i x + B_i u) \quad (15)$$

It can be noted that because there are two entries with same uncertainties in the *B* matrix, in some vertices the values of δ_i are equal. For example, $\delta_1 = \delta_9$, $\delta_2 = \delta_{10}$, … , $\delta_8 = \delta_{16}$.

Remark 1: (15) is in a quasi LPV form of coronavirus nonlinear model. It means that the presented LPV model, has same behavior as the nonlinear model. So, we can implement our controller which is designed in the next section, on both LPV and nonlinear models.

B.Controller design

In this section, we used the LPV model (15) to design a state feedback controller for the proposed model. The objective of this controller is the convergence of states *E*, *I* and *V* to zero and *R* to a positive amount while state *S* remain equal to its initial value (S_0). The reason why state *S* is a constant value is that the presented control inputs don't immune people of society for their lifetime and they may get infected any time even when they got infected once and recovered.

In the following theorem, we developed an optimal control problem in terms of some LMIs that its objective function is to minimize the cost function in the worst-case scenario of our uncertain model. Meanwhile, we considered some disturbances for the first and the second equations of (3) with positive sign, representing some specific actions like meetings or parties which are against social distancing and in the result of these kinds of actions, the number of *E* and *I* states increases. The uncertain model with disturbances is as:

$$\dot{x} = A(\delta)x + B(\delta)u + B_w w \ , B_w = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad (16)$$

So, our optimal control problem has two subjects. The controller which is considered here is a feedback controller $u = Kx$.

Theorem 1 (Optimal control design): Consider the COVID19 model as (15), if the following problem has a feasible solution:

$$\begin{cases} \max\limits_{X,M} trace(X) \\ subject\ to: \\ \begin{bmatrix} h_1 & X & M^T \\ X & Q^{-1} & 0 \\ M & 0 & R^{-1} \end{bmatrix} > 0 \\ \begin{bmatrix} h_2 & B_w & C^T \\ B_w^T & -\gamma I & 0 \\ C & 0 & -\gamma I \end{bmatrix} < 0 \end{cases} \tag{17}$$

Where $h_1 = -(A_iX + B_iM)^T - (A_iX + B_iM)$, $h_2 = (A_iX + B_iM)^T + (A_iX + B_iM)$.

Then, the $H\infty$ norm of the COVID19 model uncertainties effect on the output (the number of exposed and infected people) will be less than γ and the following cost function is minimized:

$$J = \int_0^{100} x^T Qx + u^T R_u u \tag{18}$$

Where Q and R_u are positive semi-definite matrices for finite-horizon case.

Proof:

This theorem has two parts, one part is the first LMI in (17) which guarantees the optimality of the controller and the second LMI guarantees the $H\infty$ performance of the system.

To prove the stability of the model, assume $V = x^T Px > 0$ as a common Lyapunov function and $u = Kx$ as the feedback controller. So, one has:

$$\dot{V} = \dot{x}^T Px + x^T P\dot{x} = \sum_{i=1}^{16} \delta_i(x^T[(A_i + B_iK)^T P + P(A_i + B_iK)]x) < 0 \rightarrow (A_i + B_iK)^T P + P(A_i + B_iK) < 0 \tag{19}$$

This inequality is not in LMI form, so we have to use a "change in variable and congruence trick" to turn (19) into LMI form as:

$$\xrightarrow{X=P^{-1},KX=M} A_iX + XA_i^{\ T} + B_iM + M^T B_i^{\ T} \leq 0 \tag{20}$$

To prove the first LMI in (17), considering (18) as the cost function and the following Riccati inequality [31], the solution of our problem is as follows. Note that x_0 is the initial value of x.:

$$A^T\tilde{P} + \tilde{P}A - K^T R_u K + Q < 0\ ,\ J = \int_0^{100} x^T Qx + u^T R_u u \xrightarrow{u=Kx} J = x_0^T \tilde{P} x_0 \tag{21}$$

Where $K = R_u^{\ -1} B^T \tilde{P}$. Then, the controller which minimizes our cost function can be calculated by solving the following inequality:

$$\min_{P,K}\ trace(P)$$

$$subject\ to: (A_i + B_iK)^T P + P(A_i + B_iK) - K^T R_u K + Q < 0\ , A_{cl} = A_i + B_iK \tag{22}$$

By using "change in variable and congruence trick" and "Schur complement theorem" twice as below, (22) turns to:

$$\xrightarrow{X=P^{-1}} -XA_i^{\ T} - A_iX + X^T K^T R_u KX - X^T QX > 0$$

$$\xrightarrow{Schur} \begin{bmatrix} -XA_{cl}^{\ T} - A_{cl}X + X^T K^T R_u KX & X^T \\ X & Q^{-1} \end{bmatrix} > 0$$

$$\xrightarrow{Schur\ ,KX=M} \begin{bmatrix} h_3 & X & M^T \\ X & Q^{-1} & 0 \\ M & 0 & R_u^{\ -1} \end{bmatrix} > 0 \tag{23}$$

Where $h_3 = -(A_iX + B_iM)^T - (A_iX + B_iM)$.

The proposed cost function is minimized if and only if (23) has a feasible solution.

To prove the second LMI, we allude the idea of Bounded Real Lemma (BRL) [32] which assures $\|T\|_\infty < \gamma$ where T is the transfer function of model from uncertainties disturbance to the system output, as:

$$\dot{V} + y^T y - \gamma^2 w^T w < 0 \rightarrow \sum_{i=1}^{16} \delta_i\{x^T[(A_i + B_iK)^T P + P(A_i + B_iK)]x + (x^T PB_w w) + (w^T B_w^T Px) + (x^T C^T Cx) - (\gamma^2 w^T w)\} < 0 \tag{24}$$

It can be noted that the above inequality is not in LMI form. Again, by use of "change in variable and congruence trick" as (19) and "Schur complement theorem" twice, one has:

$$\xrightarrow{Schur\ ,X=P^{-1},KX=M} \sum_{i=1}^{16} \delta_i \begin{bmatrix} Px \\ w \end{bmatrix}^T \begin{bmatrix} h_4 & B_w \\ B_w^T & -\gamma^2 I \end{bmatrix} \begin{bmatrix} Px \\ w \end{bmatrix}$$

$$\xrightarrow{Schur} \begin{cases} \min \gamma \\ subject\ to\ X > 0\ , \\ \begin{bmatrix} h & B_w & XC^T \\ B_w^T & -\gamma I & 0 \\ CX & 0 & -\gamma I \end{bmatrix} < 0 \end{cases} \tag{25}$$

Where $h_4 = XA_i^{\ T} + A_iX + M^T B_i^{\ T} + B_iM + XC^T CX, h = A_iX + XA_i^{\ T} + B_iM + M^T B_i^{\ T}$.

The disturbances of the system could be attenuated if and only if (25) has a feasible solution.

Now, both subjects of our optimal control problem turn to their final LMI form and our proof is completed. ■

The first inequality in (17) contains 16 LMIs and the second one is also containing 16 LMI and with X being positive definite $X > 0$, there are 33 LMIs, totally.

4.Numerical simulation

In this section, we simulate the results of our control design problem which were studied in the previous section to find out how it can affect preventing faster disease spread, using YALMIP toolbox [33]. To shortly look through the steps which had taken to this stage, first, we simplified the nonlinear model by assuming the state S as a constant value. Then, we linearized the simplified model (3) by the Jacobian method. The polytopic LPV model is extracted considering some uncertainties in the model, in the next stage. Finally, a feedback control configuration is exploited to design a proper controller for the LPV model. As we said before, because the presented control inputs don't immune people of society for their lifetime and they may get infected again, we can suppose that the number of susceptible individuals is a constant number S_0 which is its initial condition. For simulation, we need the initial conditions of our states. For this purpose, we used [34] which presents the real data of Wuhan City in China from January 2020 to February 2020 (in this period, the city was quarantined). So, the initial values of states are reported as:

$$(E(0), I(0), R(0), V(0)) = (1000,475,10,10000) \quad (26)$$

In simulating the presented problem, we considered three different cases:

1. The first scenario is considering open-loop response of the model, meaning that $u_1 = u_2 = u_3 = 0$. Our goal in this scenario is to demonstrate the response of the model without any control action.
2. The second scenario is to analyze the closed-loop feedback controller response, meaning that $u_1, u_2, u_3 \neq 0$. In this case, the goal is to stabilize the model without using the controller designed in theorem 1.
3. The third scenario is to simulate the final control configuration with two objectives which were designed in theorem 1.

We assume that some parameters include some uncertainties which contain E, I, and V states and the parameter ω and their intervals are presented in Table II.

TABLE II. VALUES OF INTERVALS OF THE STATES WHICH UNCERTAIN PARAMETERS CONSIST OF THEM

parameter	Definition	value
$(\underline{E}, \bar{E})$	(Minimum of state E, Maximum of state E)	(500,1500)
$(\underline{I}, \bar{I})$	(Minimum of state I, Maximum of state I)	(400,500)
$(\underline{V}, \bar{V})$	(Minimum of state V, Maximum of state V)	(9000,11000)
$(\underline{\omega}, \bar{\omega})$	(Minimum of parameter ω, Maximum of parameter ω)	(0.01,0.015)

We also considered R_u, Q matrices as:

R_u =[2 0 0;0 1 0;0 0 1],

Q =[2 0 0 0;0 2 0 0;0 0 1 0;0 0 0 1]. (27)

The parameters ω could vary in the interval which is presented on Table II and its changing profile is also showed in "Fig. 2".

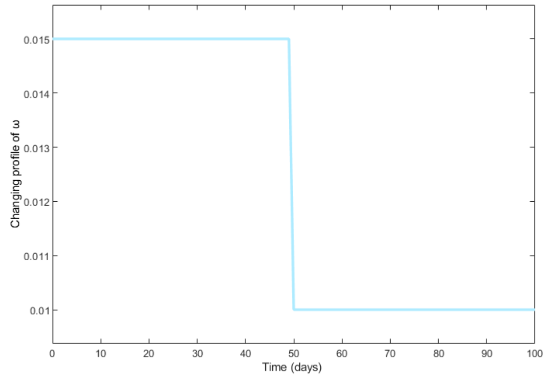


Figure 2. The changing profile of the uncertain parameter $\boldsymbol{\omega}$

A.First case scenario

Our first scenario is to remove control inputs to monitor the rate of disease transmission and see what will happen to the society if the three proposed intervention strategies have not been used.

As it can be observed from "Fig. 3" the number of infected and exposed individuals converge to zero after almost 90 and 40 days, respectively. The number of recovered individuals is also converging to a steady-state positive value and the density of coronavirus in the environment reaches to zero. So, the proposed model for coronavirus transmission in (3) is stable by itself.

"Fig. 4" shows that the transmission rates between susceptible individuals and exposed ($\beta_E(E)$) and infected individuals ($\beta_I(I)$) and the environment ($\beta_V(V)$), are increasing and converge to a steady-state value in all of them, and "Fig. 5" shows the basic reproduction number (R_0) value, in case we ignore the control inputs, is constant and equals to 4.1835. Obviously, the model is stable by itself, but the R_0 value is not in the desired range (less than 1).

B.Second case scenario

In this case, we used the feedback controller designed in the previous section to analyze the effects of the controller on the proposed model of coronavirus transmission. The designed feedback controller's gain is calculated as:

$$K = \begin{bmatrix} -0.0018 & -0.0007 & -0.0004 & -0.0002 \\ -0.0001 & 0.0001 & -0.0009 & -0.0004 \\ -0.0002 & 0 & 0 & 0 \end{bmatrix} \quad (28)$$

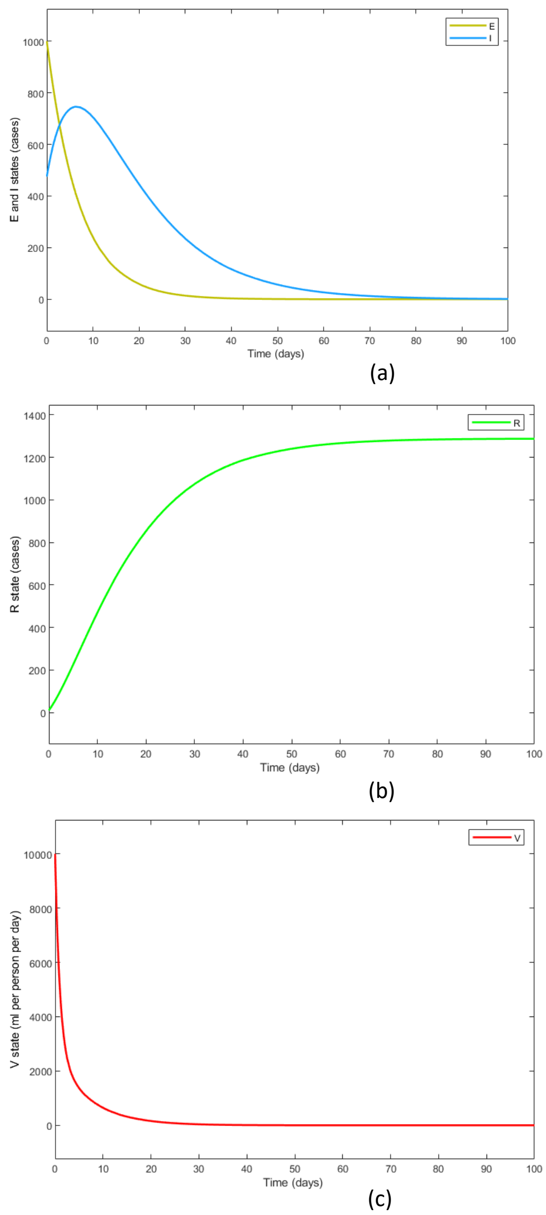


Figure 3. The results of simulation of uncontrolled coronavirus transmission model in 100-day period of time. a) Exposed and infected individuals (cases). b) Recovered individuals(cases). c) The density of coronavirus in the environment (ml per person per days).

“Fig. 6” shows that the basic reproduction number (R_0) graph which was expressed as a nonlinear function in (5). It can be inferred from “Fig. 6” that the virus is very contagious in very early stages, but its contagiousness could be lowered if we use suitable control interventions like isolation and quarantine, good medical care, and disinfecting the surfaces. The results of using these methods can be seen where finally R_0 almost converges to two which is a tangible change to secure more people of the society. It can be noticed that we cannot reduce the coronavirus contagiousness lower than one without vaccination.

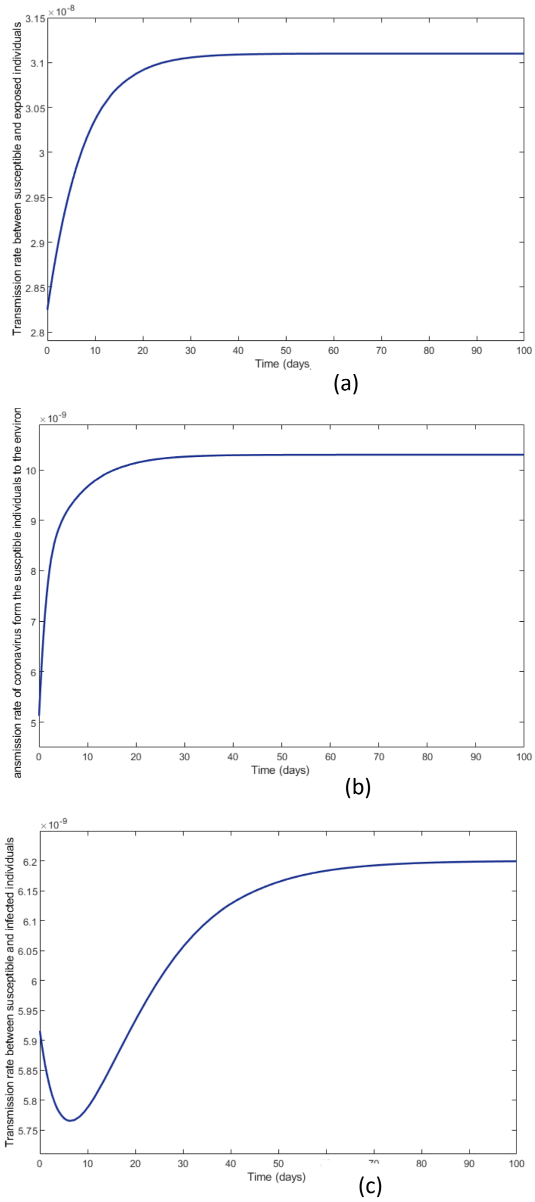


Figure 4.The nonlinear transmission rate between a) Susceptible and exposed individuals b) Susceptible and infected individuals c) Susceptible individuals and environment (in case u1=u2=u3=0)

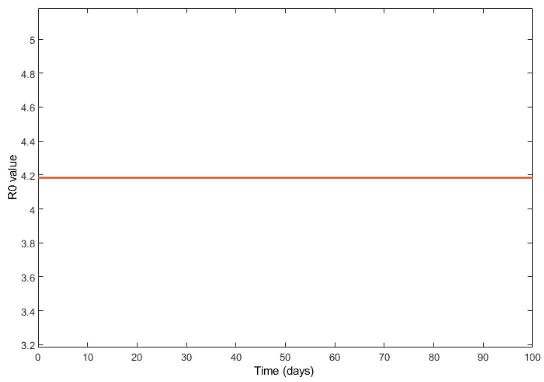


Figure 5. The reproduction number (R_0) value when three control inputs $u_1=u_2=u_3=0$

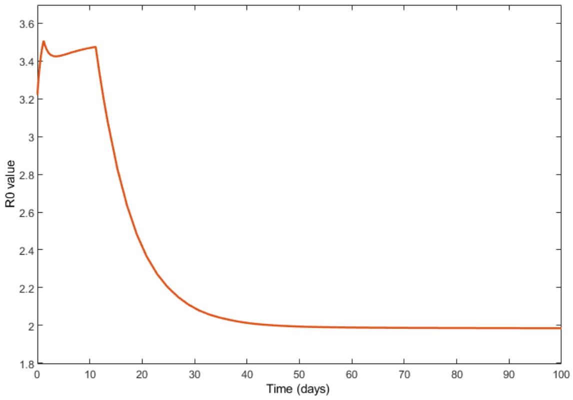


Figure 6. The reproduction number (R_0) value when three control inputs u_1, u_2, $u_3 \neq 0$

Pondering "Fig. 7" leads us to the fact that this controller can reduce the amount of infected and exposed individuals, to some extent. We can prevent the disease from killing and infecting many people and save their lives by using good control strategies in the proper stage of coronavirus transmission. The graph of recovered individuals and density of coronavirus in the environment is almost similar, but a bit faster than the previous part. It means that the controller makes the process of healing and disinfection faster.

As it can be observed from "Fig. 8", control inputs are applied on the LPV model of coronavirus transmission. As expected, the first control input u_1 should be maximum in the early stage of the process of controlling virus transmission because the susceptible individuals don't get immune to the disease without vaccination. So, all the people of the society should respect isolation, quarantine, and lockdown rules especially at the early stages, and then the rate of implementing the preventive actions can be more relaxed until the end of the controlling period.

We can also see form "Fig. 8", that u_2 is always in its maximum value and the reason is that we need maximum recovery rate at any stage of disease spread because the sources for this control action are limited. The third control input u_3 is converging to a value near zero fast, because in an idealistic scenario in which all people respect all presented control strategies completely, the density of coronavirus in the environment could be lowered as minimum as possible.

"Fig. 9" shows that the same as the previous scenario, the rates of transmission increase and then converge to a steady-state value. But in comparison to the first scenario, it can be observed that the rate of transmission, especially for infected individuals, is converging faster.

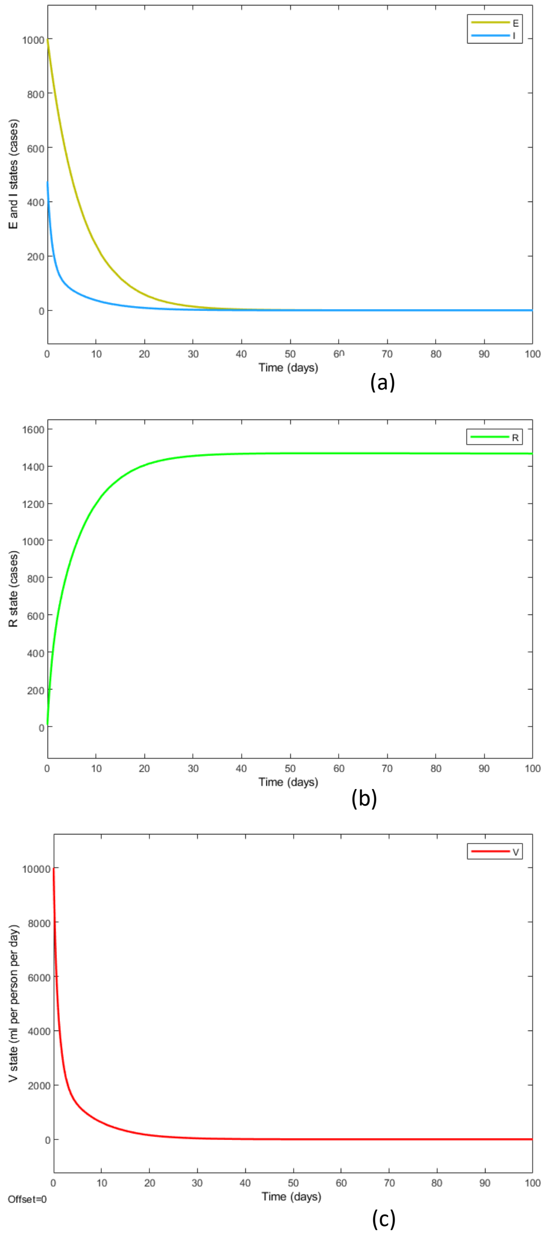


Figure 7. The results of simulation of controlled coronavirus transmission model in 100-day period of time. a) Exposed and infected individuals (cases). b) Recovered individuals (cases). c) The density of coronavirus in the environment (ml per person per days).

C.Third case scenario

In this scenario, again we used the feedback controller and form an optimal control problem with two subjects. This case led us to more realistic results because we considered a cost function for our problem. The designed feedback controller's gain and the parameter γ are calculated as:

$$K = \begin{bmatrix} -0.4183 & -1.4510 & -1.2919 & -0.0030 \\ 0.02500 & 0.0884 & 0.0745 & 0.0002 \\ 0.0001 & 0.0010 & 0.0009 & -0.0009 \end{bmatrix}, \quad \gamma = 37.4337 \tag{29}$$

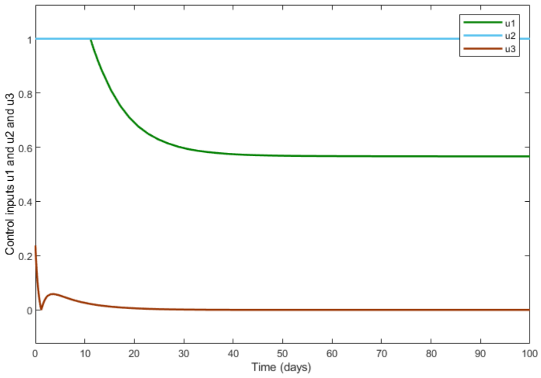
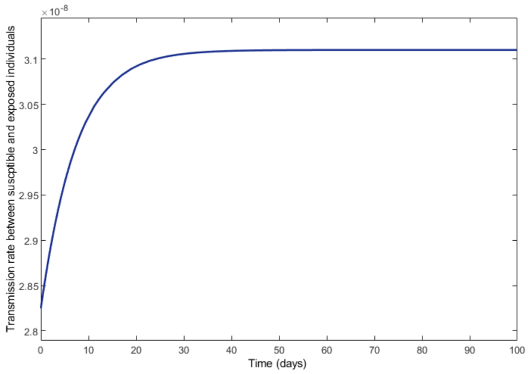
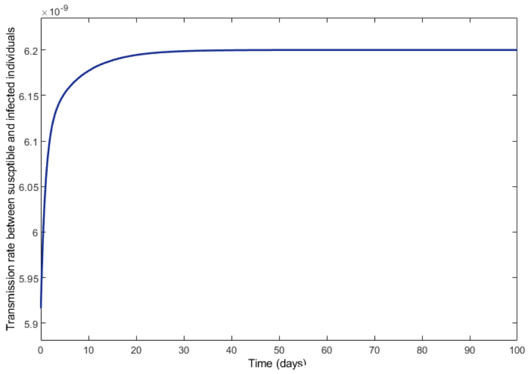


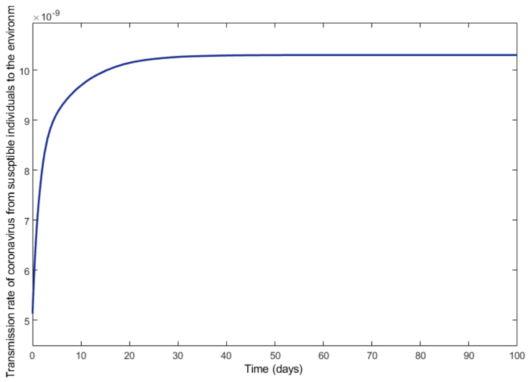
Figure 8. control input u_1 (preventive actions), u_2 (good medical care) and u_3 (disinfection actions)



(a)



(b)



(c)

Figure 9. The nonlinear transmission rate between a) Susceptible and exposed individuals b) Susceptible and infected individuals c) Susceptible individuals and environment (in case u_1, u_2, $u_3 \neq 0$)

As it can be perceived from "Fig. 10", the R_0 value in this case finally converges to 3.6 which is a higher amount than the previous case and it is because the value of control inputs converge to a constant value (because of the existence of cost function). So, the value of R_0 increases to some extent, but it still is much lower than the first scenario. It can be noted that the sudden increase in the middle of simulation period for R_0 is because of the changing profile of the uncertain parameter ω which was presented in "Fig. 2".

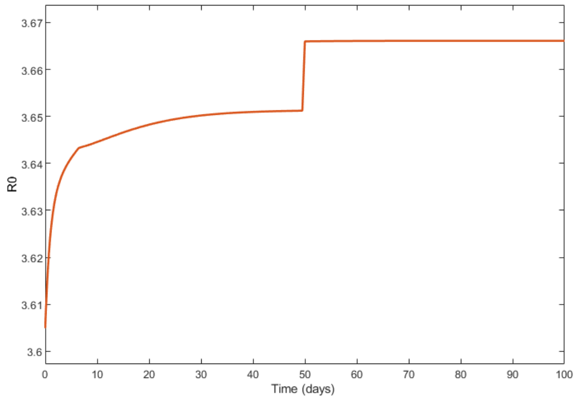


Figure 10.The reproduction number (R0) value when there are some constraints on three control inputs

It is noticeable from "Fig. 11" that the number of infected and exposed individuals and also the density of coronavirus in the environment converge to their equilibrium point properly. The results in this case are similar to the first case, but the convergence of states is faster because we optimized the control inputs' values. So, the optimal control acted suitably, even with the existence of disturbances and a cost function, it leads the system to almost same results.

"Fig. 12" also shows u_1 has almost its maximum value for the whole simulation period because the value of exposed individuals is a very high amount in early stages and this is our most practical control action. As the number of susceptible individuals is a constant value, we should have u_1 with its maximum value to prevent more virus transmission and a greater number of infected individuals. The value of u_2 converges to a fixed value which is needed always to secure people in the whole simulation period and with reduction in the number of infected individuals in early stages. In comparison with the last scenario, we can obviously see the effect of the cost function on the value of control inputs. The third control input u_3 has reduced after decreasing the high value of concentration of coronavirus in the environment in early stages. Then, with reducing the amount of coronavirus concentration and because of the cost function, the value of this controller converges to a very small value. According to the weights in (27), the values of third and second controllers are much less than the first controller because the main controlling measure for preventing coronavirus transmission is the first one.

"Fig. 13" also shows that the same as the second scenario, the transmission rate between different individuals and the environment converge to some positive steady-state values, but they are a bit slower than the second scenario.

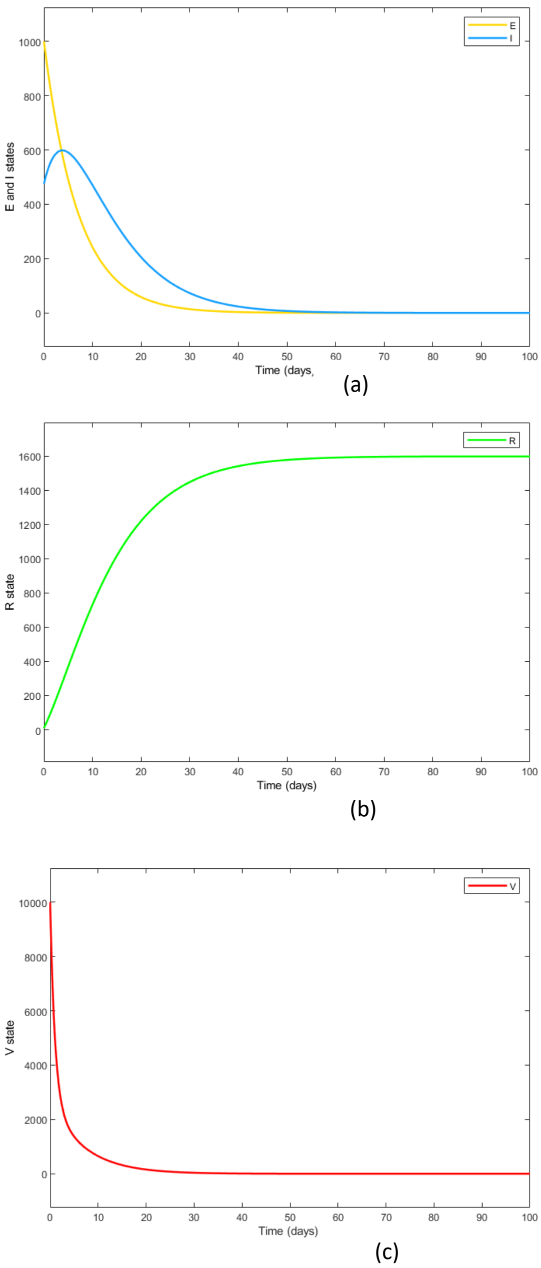


Figure 11.The results of simulation of optimal control on coronavirus transmission model in 100-day period of time. a) Exposed and infected individuals (cases). b) Recovered individuals (cases). c) The density of coronavirus in the environment (ml per person per day)

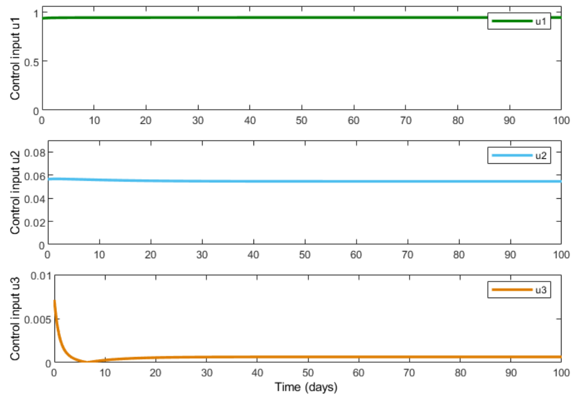


Figure 12.control input u1 (preventive actions), u2 (good medical care) and u3 (disinfection actions) for optimal control problem

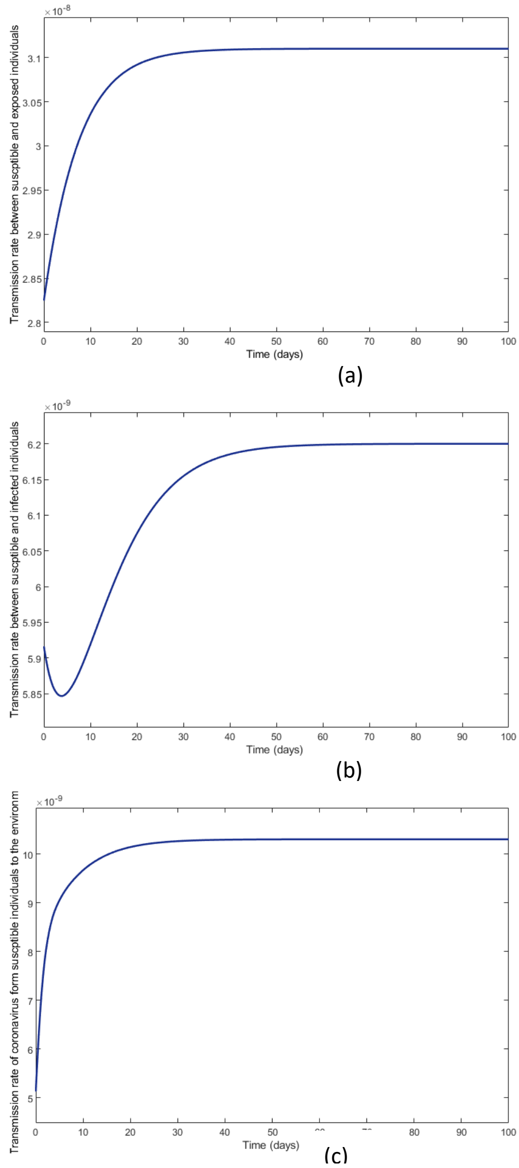


Figure 13. The nonlinear transmission rate between a) Susceptible and exposed individuals b) Susceptible and infected individuals c) Susceptible individuals and environment (in optimal control case)

5.Conclusion

In this paper, we presented a feedback controller configuration for a polytopic LPV model of coronavirus transmission of Wuhan City which was the origin of coronavirus spread. The final goal of designing this controller is to reduce the number of infected and exposed individuals, while lowering coronavirus concentration from the environment around us, as minimum as possible. An optimal LPV robust controller is designed to control covid-19 spread as fast as possible due to a defined cost function. Furthermore, we used a nonlinear expression for basic reproduction number (R_0), based on control inputs to see and analyze how the controller affect the contagiousness of the disease. We observed that by using these intervention strategies, the number of infected and exposed infected individuals converge to zero faster, while the number of recovered individuals converge a positive steady-state value and the density of coronavirus in the environment converges to zero. These results indicate that if the control intervention measures were implemented in a suitable time (maybe as fast as possible) and the sources of control methods were sufficient and available enough, more people’s lives could be saved and more people get temporary immunity to the disease. The results also demonstrated that the proposed controller is robust against a wide range of uncertainties. In conclusion, based on available sources of different control inputs, we can choose second or third scenarios to control virus transmission in the best way possible.

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