Robust State-Feedback Control of Uncertain LPV systems Using Integral Quadratic Constraints

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***Abstract*—A linear matrix inequality (LMI)-based algorithm is developed to design a robust state-feedback controller using integral quadratic constraints (IQCs) for an uncertain linear parameter varying system (LPVS). The uncertain LPVS is described by an interconnection of a nominal LPVS which is solely dependent on the measurable parameters and a block-structured uncertainty. The IQC approach is implemented to model the input/output behavior of the uncertainties. In general, the robust synthesis method and the IQC stability analysis for the uncertain LPVS lead to a non-convex problem and are solved by the iterative algorithms. However, in the proposed method, the problem is converted into a convex problem. Therefore, the LPV synthesis for the nominal LPVS and the IQC analysis for handling uncertainties are performed simultaneously. Consequently, without any constraints on nominal system matrices, the proposed method might achieve a better performance and less computational burden. Furthermore, the object is to minimize the l_2-gain, H_∞ control, when the closed-loop asymptotical stability is also guaranteed. The performance and effectiveness of the proposed method are demonstrated based on two examples.**

***Keywords*: gain-scheduled controller, integral quadratic constraints, polytopic system, uncertain linear parameter varying systems.**

I. Introduction

LPVS are a class of linear systems whose system matrices depend on time-varying parameters and are described as nominal or uncertain systems by considering the measurability of the parameters [1],[2]. In most practical systems, some parameters cannot be measured or measurement is not cost-effective for any reason. Consequently, they should be considered as an uncertain LPVS [3]. The control of an uncertain LPVS is still a challenging problem in control theory because both robust stability and desired performance must be guaranteed simultaneously. In [4], a general framework for the uncertain LPVS has been proposed in order to design a full order gain-scheduling controller based on the linear fractional transformation (LFT) where the uncertain LPVS is decomposed into a known linear time-invariant system with a block-structured uncertainty as a standard LFT interconnection. The method suggested in [4] has several limitations: a) the system should be converted into a general LFT representation; b) the proposed algorithm will be conservative because the parameters and uncertainties are assumed as an uncertainty in the design procedure. However, in this paper using the concept of IQC, and considering the H_∞ performance proposes a gain-scheduled controller in order to reduce conservatism in which measurable parameters will be entered in the controller structure as shown in Fig. 1. Fig. 1 illustrates the synthesis problem of the gain-scheduled H_∞ controller for the uncertain LPVS to obtain parameter-dependent $K\big(\theta_1(t)\big)$, where $\theta_1(t)$ is measurable, so that the closed-loop stability is satisfied for all $\theta_2(t)$ and Δ, while the induced l_2-gain from the noise input $d(t)$ to the controlled output $e(t)$ is minimized [4].

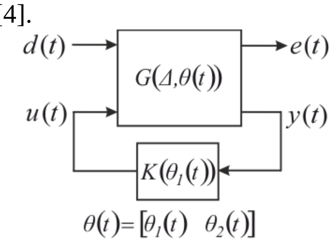

Fig. 1. Synthesis interconnection for uncertain LPVSs in which $\theta_2(t)$ and Δ are the unmeasurable parameter and the model uncertainty, respectively.

In recent years, the H_∞ controller synthesis for uncertain LPVS has been considered by the IQC concept. The IQC approaches describe input/output behavior of the uncertainties (for example parametric and dynamic uncertainties, nonlinear components such as delay and dead-zone in [5]) and can be expressed in time and frequency domains [6]. Also, it provides a general framework for robust synthesis and performance analysis of an uncertain system [7]–[10]. As noted in [3], a hard IQC that its integral constraints are valid over a finite-time interval with the time domain interpretation should be applied to obtain gain-scheduled controller for the uncertain LPVSs because they do not have a valid frequency response. By using the IQC, the controller is given by iteration of two steps [11]–[16]; while in general, this problem leads to a non-convex problem [3]. First the design step, in this step a gain-scheduled controller for the nominal LPVS (no uncertainty and no unmeasurable parameters) is designed based on the algorithm proposed in [17]–[21]. Second the analysis step, here the robust stability and performance are investigated for a designed controller in the previous step by the IQC theorem suggested in [22]. These two steps can also be done by heuristic methods. For instance, an alternative algorithm has been proposed in [2] which is similar to the well-known DK-iteration for μ synthesis [23] to follow the iterations of these two steps. The iterative methods given in [11]–[16] have advantages and disadvantages in comparison with the LFT based technique in [4]: a) The iterative methods can achieve less conservative results; because all the parameters are assumed to be uncertain in LFT techniques. b) In the iterative methods, controller design and robust stability are done in two steps; but in the LFT method, these done simultaneously. c) LFT methods use parameters and therefore, the controller will be in gain-scheduled form; in which an LTI controller is given in iterative methods. Furthermore, both methods can be used to design both state feedback and output feedback controllers.

In this paper, an LMI-based method is proposed to design a gain-based H_∞ controller using IQC for an uncertain polytopic LPVS. The proposed method has advantages and disadvantages compared to previous methods: a) Both LPV controller design and robust stability analysis are considered simultaneously. However, the methods shown in [11]–[16] use two steps and thus, increase conservativeness. b) The proposed method might achieve a better performance (less conservative results) with less computational burden in comparison with the suggested methods in [3], [4] because only the unmeasurable parameters are defined as uncertainty. c) In the algorithm presented in [11]–[16], the design problem leads to a non-convex problem, but in this paper, a change variable is defined to transform the non-convex problem into a convex problem. However, the range of parameters is assumed to be polytopic, which is a limitation. d) By assuming the state variables are available, the LMI-based algorithm is proposed without any constraints on nominal system matrices to derive the robust gain-scheduling controller, which is the main novelty of this paper. e) As an extension of the main novelty, a gain-scheduled controller is proposed to guarantee the maximum stability margin against a class of uncertainties. In addition, the fixed state-feedback controllers in every vertex of available parameters will be calculated as off-line, even though they are interpolated in real-time by the measurable parameters.

The paper is organized as follows: Notation and background of both IQC concept and robustness analysis of uncertain LPVSs using IQCs are described in Section II. The LMI-based algorithm for designing the robust state-feedback controller is introduced in Section III. Simulation results with two numerical examples are presented in Section V. Lastly, Section VI draws the conclusion.

II. Background

A. Notation

L_{2e} indicates an extended space of vector-valued locally square integrable on finite intervals (i.e. on all intervals$[0\ T], T \geq 0$), whereas $L_2 \subset L_{2e}$ is a signal subspace with limited energy. In symmetric matrices, the symbol $(*)$ shows a term which must be substituted by symmetry. I, $diag(\ldots)$, and $He(F)$ indicate the identity matrix with the appropriate dimension, a block diagonal matrix, and $F+F^T$, respectively.

B. Integral Quadratic Constraints

Fig. 2 shows the uncertain LPVS where T is the nominal LPVS and Δ is a bounded casual operator relating $v(t)$ and $w(t)$. The perturbation Δ can describe a wide variety of nonlinear elements and uncertainties, e.g. saturation, delay and norm-bounded uncertainties. The signals $v(t) \in L_{2e}^{q_z}[0\ \infty)$ and $w(t) \in L_{2e}^{f_w}[0\ \infty)$ satisfy the frequency domain IQC with a measurable hermitian matrix $\Pi(j\omega)$ called IQC multiplier, if:

$$\int_{-\infty}^{\infty}\begin{bmatrix}\hat{V}(jw)\\ \widehat{W}(jw)\end{bmatrix}^{*}\Pi(jw)\begin{bmatrix}\hat{V}(jw)\\ \widehat{W}(jw)\end{bmatrix}dw\geq 0 \qquad (1)$$

where $\hat{V}(j\omega)$, $\widehat{W}(j\omega)$ are the Fourier transformations of $v(t)$, $w(t)$ respectively. The frequency domain inequality (1) is also represented in time domain form via a non-unique factorization $\Pi(j\omega)=\psi^{*}(j\omega)\ M\,\psi(j\omega)$ referred by a pair (ψ,M) in which M is a constant matrix and ψ is a stable system with $v(t),w(t)$ as inputs [6]. The perturbation Δ satisfies the time domain IQC with (ψ,M) showed in Fig. 3, if:

$$\int_{0}^{T}z^{T}(t)\,M\,z(t)\,dt\geq 0 \qquad (2)$$

where $z(t)$ is the output of ψ with zero initial conditions. Also, the matrix M can be partitioned as:

$$M=\begin{bmatrix}\Pi_{11} & \Pi_{12}\\ * & \Pi_{22}\end{bmatrix},\Pi_{11}>0,\Pi_{22}<0 \qquad (3)$$

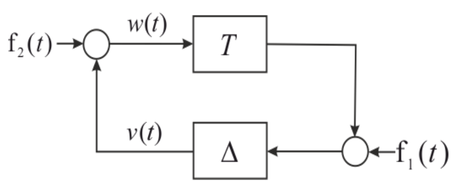

Fig. 2. IQC feedback interconnection

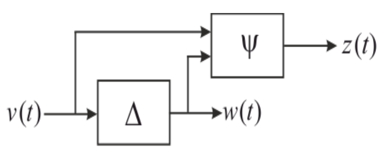

Fig. 3. Graphical time domain IQC

Time domain inequality (2) is called the hard IQC. In contrast, if this inequality is held for $T=\infty$ it is referred as the soft IQC. The time domain and hard IQCs are used in this paper because the LPVSs do not have a valid frequency domain response and hence the dissipation inequalities will be used to obtain sufficient stability conditions [22].

C. Robustness Analysis of Uncertain LPVSs Using IQCs

In this section, the robustness analysis of the uncertain LPVS is given in which the uncertainties and unmeasurable parameters are defined by IQCs. Fig. 4 shows the nominal LPVS T and the perturbation Δ that satisfies the time domain IQC (ψ,M), in which the state-space realization is given by:

$$\begin{aligned}\dot{x}(t)&=A_{cl}\left(\theta(t)\right)x(t)+B_{1cl}\left(\theta(t)\right)w(t)\\&\quad+B_{2cl}\left(\theta(t)\right)d(t)\\ z(t)&=C_{1cl}\left(\theta(t)\right)x(t)+D_{11cl}\left(\theta(t)\right)w(t)\\&\quad+D_{12cl}\left(\theta(t)\right)d(t)\\ e(t)&=C_{2cl}\left(\theta(t)\right)x(t)+D_{21cl}\left(\theta(t)\right)w(t)\\&\quad+D_{22cl}\left(\theta(t)\right)d(t)\end{aligned} \qquad (4)$$

where the nominal LPVS depends on the measurable parameters $\theta(t)$ specified as $\theta(t)\in\Theta$ in which Θ is a polytope set. The desired performance of the closed-loop system (4) is defined the minimizing of H_∞ performance $\|F_u(T,\Delta)\|_\infty=sup_{d(t)\in L_2}\|e(t)\|_2/\|d(t)\|_2$ for all $\theta(t)\in\Theta$ and $\Delta\in IQC(\psi,M)$, or equivalently:

$$\sup_{\Delta\in IQC(\psi,M),\theta\in\Theta}\|F_u(T,\Delta)\|_\infty<\gamma \qquad (5)$$

In the following, the time-varying parameter $\theta(t)$ is indicated by θ to shorten the notation. Theorem 1 presents the sufficient conditions for calculating the robust performance level γ.

Theorem 1: The LPVS (4) is exponentially stable and $\|F_u(T,\Delta)\|_\infty<\gamma$ for $\Delta\in IQC(\psi,M)$ if the symmetric matrix $P>0$ and positive scalars λ_1, λ_2, and γ are exist such that the following inequality is feasible for all $\theta\in\Theta$.

$$\begin{aligned}&\begin{bmatrix}He\left(PA_{cl}(\theta)\right) & * & *\\ B_{1cl}^{T}(\theta)P & 0 & *\\ B_{2cl}^{T}(\theta)P & 0 & -\lambda_1\gamma I\end{bmatrix}\\&+\frac{\lambda_1}{\gamma}\begin{bmatrix}C_{2cl}^{T}(\theta)\\ D_{21cl}^{T}(\theta)\\ D_{22cl}^{T}(\theta)\end{bmatrix}\left[C_{2cl}(\theta)\quad D_{21cl}(\theta)\quad D_{22cl}(\theta)\right]\\&+\lambda_2\begin{bmatrix}C_{1cl}^{T}(\theta)\\ D_{11cl}^{T}(\theta)\\ D_{12cl}^{T}(\theta)\end{bmatrix}M\left[C_{1cl}(\theta)\quad D_{11cl}(\theta)\quad D_{12cl}(\theta)\right]<0\end{aligned} \qquad (6)$$

Proof: The proof is based on the dissipation inequality by defining the storage function $V(t)=x^{T}(t)Px(t)\geq 0$. By multiplying both sides of equation (6) by $[x^{T}(t),w^{T}(t),d^{T}(t)]$ and its transpose respectively, the inequality (6) will be:

$$\begin{aligned}&He\left(x^{T}(t)\,P\,A_{cl}(\theta)x(t)+x^{T}(t)\,PB_{cl}(\theta)\,d(t)\right)\\&+\lambda_1\gamma^{-1}e^{T}(t)e(t)-\lambda_1\gamma d^{T}(t)d(t)+\lambda_2 z^{T}(t)Mz(t)<0\end{aligned} \qquad (7)$$

or equivalently:

$$\begin{aligned}&\dot{V}(t)+\lambda_1\gamma^{-1}e^{T}(t)e(t)\\&-\lambda_1\gamma d^{T}(t)d(t)+\lambda_2\,z^{T}(t)Mz(t)<0\end{aligned} \qquad (8)$$

where $\dot{V}(t)=\dot{x}^{T}(t)P\,x(t)+x^{T}(t)P\dot{x}^{T}(t)$. Integrating equation (8) over the interval $[0,T]$ with $x(0)=0$ results in:

$$\begin{aligned}&V(T)+\lambda_1\gamma^{-1}\int_{0}^{T}e^{T}(t)e(t)dt-\lambda_1\gamma\int_{0}^{T}d^{T}(t)d(t)\,dt\\&\qquad+\lambda_2\int_{0}^{T}z^{T}(t)Mz(t)\,dt<0\end{aligned} \qquad (9)$$

$V(T)\geq 0$ and $\int_{0}^{T}z^{T}(t)Mz(t)\,dt>0$ imply that above inequality holds if:

$$\int_{0}^{T}e^{T}(t)e(t)dt<\gamma^{2}\int_{0}^{T}d^{T}(t)d(t)\,dt \qquad (10)$$

for all $T\geq 0$ or equivalently:

$$\|T\|_\infty:=\frac{\|e(t)\|_2}{\|d(t)\|_2}<\gamma \qquad (11)$$

Therefore, the proof is done. ∎

Theorem 1 illustrates an extension of the bounded real lemma (BRL) in which the scalar parameters λ_1 and λ_2 are degrees of freedom. Moreover, if λ_1 is selected as $1/\gamma$ or 1, the used BRL in references [22] and [24] will be resulted, respectively. Two details should be considered in theorem 1. First, the inequality (6) leads to an infinite collection of inequalities because it is parameter-dependent that must be satisfied for $\theta\in\Theta$. Converting the infinite inequalities into the finite number of conditions can be done by using the gridding [17], [19] or the polytopic methods [25], [26]. In the gridding technique, the parameter set is gridded to the finite number of points and then the inequalities are checked at these points (no all parameters); however, it can be employed for any form of the parameter set. Hence, these methods satisfy the local stability and performance. Nevertheless, the polytopic technique which involves a convex parameter set guarantees both the global stability and performance for all $\theta \in \Theta$. The second detail is that the Lyapunov matrix P can be assumed parameter-dependent. In this paper, the polytopic method has been employed and hence, the constant matrix P is also used and the following definition will be required.

Definition 1 [27], [28]*:* If the parameters change in a polytope set can be obtained in every time by:

$$\theta \in \Theta := Co\{N_1, N_2, \dots, N_r\} = \sum_{i=1}^{r} \alpha_i N_i, \sum_{i=1}^{r} \alpha_i = 1, \alpha_i \geq 0 \tag{12}$$

where N_i is the ith vertex value of the polytope set. Also, the LPVS (4) is polytopic if the parameters change in a polytope and the system matrices can be derived by:

$$\begin{pmatrix} A_{cl}(\theta) & B_{cl}(\theta) \\ C_{cl}(\theta) & D_{cl}(\theta) \end{pmatrix} \in Co\left\{\begin{pmatrix} A_i & B_i \\ C_i & D_i \end{pmatrix} : i = 1,2,\dots,r\right\}, \quad \begin{pmatrix} A_i & B_i \\ C_i & D_i \end{pmatrix} := \begin{pmatrix} A(N_i) & B(N_i) \\ C(N_i) & D(N_i) \end{pmatrix}, \theta \in \Theta \tag{13}$$

III. Robust State-Feedback Controller Synthesis

In this section, first, the problem formulation that considers the open-loop system, design objective, and problem assumptions are presented. Then, an LMI-based approach is proved to find a gain-scheduled controller that guarantees the H_∞ performance and asymptotically stability for the desired uncertain LPVS.

A. Problem Formulation

The robust synthesis problem is to design an LPV controller, $K(\theta)$ for the uncertain LPVS, G with both noise $w(t)$ and perturbation Δ which is shown in Fig. 5, considered an open-loop system described by the following state-space realization.

$$\begin{aligned} \dot{x}(t) &= A(\theta)x(t) + B_1(\theta)w(t) + B_2(\theta)d(t) + B(\theta)u(t) \\ z(t) &= C_1(\theta)x(t) + D_{11}(\theta)w(t) + D_{12}(\theta)d(t) + D_z(\theta)u(t) \\ e(t) &= C_2(\theta)x(t) + D_{21}(\theta)w(t) + D_{22}(\theta)d(t) + D_e(\theta)u(t) \\ y(t) &= x(t) \end{aligned} \tag{14}$$

where $A \in \mathcal{R}^n$, $D_z \in \mathcal{R}^{q_\Delta \times m}$, $D_{22} \in \mathcal{R}^{q_e \times f_d}$, $D_{21} \in \mathcal{R}^{q_e \times f_w}$. The system matrices and the parameter θ in (14) belong to the polytope set or:

$$\Omega = [\theta, A(\theta), B_1(\theta), B_2(\theta), B(\theta), C_1(\theta), D_{11}(\theta), D_{12}(\theta), C_2(\theta), D_{21}(\theta), D_{22}(\theta), D_e(\theta)] = \sum_{i=1}^{r} \alpha_i [\theta_i, A_i, B_{1i}, B_{2i}, B_i, C_{1i}, D_{11i}, D_{12i}, C_{2i}, D_{21i}, D_{22i}, D_{ei}], \sum_{i=1}^{r} \alpha_i = 1, \alpha_i \geq 0 \tag{15}$$

Therefore, from definition 1 the system G will be a polytopic system. The main objective of this work is to find a gain-scheduled controller

$$u(t) = K(\theta)\, y(t) = \sum_{i=1}^{r} \alpha_i K_i, \sum_{i=1}^{r} \alpha_i = 1, \alpha_i \geq 0 \tag{16}$$

where $K_i, \forall i = 1, \dots, r$ are obtained from off-line analysis, such that the closed-loop system is asymptotically stable by the consideration of disturbance Δ and also the H_∞ performance with l_2-gain γ is guaranteed, i.e.,

$$\sup_{\Delta \in IQC(\psi, M), \theta \in \Theta} \|F_u(F_L(G,K),\Delta)\|_\infty < \gamma \tag{17}$$

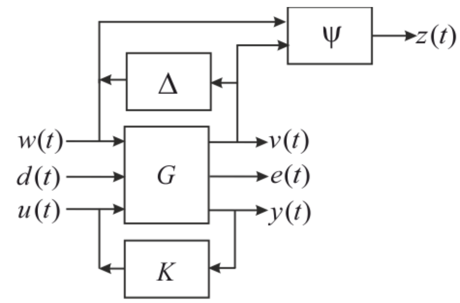

Fig. 5. Open loop system G, a perturbation Δ, and the LPV controller K

Moreover, the following assumption and lemma will be used in the next sections.

Assumption 1: The perturbation Δ is a bounded casual operator. Furthermore, the stable system $\psi(jw)$ selects an identity system, that is:

$$z(t) = \begin{bmatrix} C_{11}(\theta) & D_{111}(\theta) & D_{121}(\theta) & D_{z1}(\theta) \\ 0 & I & 0 & 0 \end{bmatrix} \times [x^T(t) \quad w^T(t) \quad d^T(t) \quad u^T(t)]^T \tag{18}$$

where $C_{11}(\theta) \in \mathcal{R}^{q_z \times n}$.

Lemma 1 (Schur complement) [29]: F has an affine dependency in terms of x as follows:

$$F(x) = \begin{pmatrix} F_{11}(x) & F_{12}(x) \\ F_{21}(x) & F_{22}(x) \end{pmatrix} < 0 \tag{19}$$

where $F_{11}(x)$ and $F_{22}(x)$ are the square matrices. $F(x)$ is negative definite if and only if:

$$\begin{cases} F_{11}(x) < 0 \\ F_{22}(x) - F_{21}(x)[F_{11}(x)]^{-1}F_{12}(x) < 0 \end{cases} \tag{20}$$

or:

$$\begin{cases} F_{22}(x) < 0 \\ F_{11}(x) - F_{12}(x)[F_{22}(x)]^{-1}F_{21}(x) < 0 \end{cases} \tag{21}$$

Remark 1: Because the final gain-scheduled controller is interpolated employing all of the measurable parameters, if a number of parameters are not measurable, they should be embedded in the perturbation Δ. In practice point of view, this issue will be a key point for designing a robust controller. Consequently, the proposed algorithm can also be applied when the unmeasurable parameters have appeared in the linear parameter varying model. Furthermore, in order to obtain a simpler controller, some available parameters can be defined as a perturbation Δ. By modeling the LPVS as an uncertain LTI system, this idea will be perfect if the desired closed-loop performance is achievable.

IV. Main Results

In this section, before the main results are given, the time-domain IQCs (ψ, M) should be determined. Assumption 1 defines ψ, and the matrix M should be chosen by considering the type of disturbance Δ listed in [5]. In the following, we assume that the matrix M is known and partitioned as equation (3).

Theorem 2: Consider the uncertain LPVS (14) in which the IQC matrices, ψ and M are defined by an identity system and partitioned as (3), respectively. For known scalar parameter λ_2, if the matrices $L_i, \forall i = 1, \dots, r$ and $Q = Q^T > 0$, and a scalar γ^2 are exist such that:

$$\begin{aligned} &\varphi_{ii} < 0 &&, i = 1,2,\dots,r \\ &\varphi_{ij} + \varphi_{ji} < 0 &&, i < j = 1,2,\dots,r \end{aligned} \tag{22}$$

where

$$\varphi_{ij} = \begin{bmatrix} He(A_iQ + B_iL_j) & * & * & * & * \\ B_{1i}^T + (\Pi'_{12}I)^T(C_{11i}Q + D_{z1i}L_j) & He(D_{111i}^T\Pi'_{12}I) + I^T\Pi'_{22}I & * & * & * \\ B_{2i}^T & D_{121i}^T\Pi'_{12}I & -\gamma^2 I & * & * \\ C_{11i}Q + D_{z1i}L_j & D_{111i} & D_{121i} & -{\Pi'_{11}}^{-1} & * \\ C_{2i}Q + D_{ei}L_j & D_{21i} & D_{22i} & 0 & -I \end{bmatrix} \tag{23}$$

$$\begin{bmatrix} \Pi'_{11} & \Pi'_{12} \\ * & \Pi'_{22} \end{bmatrix} = \lambda_2 \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix}$$

Then, the gain-scheduled controller,

$$K = \sum_{i=1}^{r} \alpha_i K_i = \sum_{i=1}^{r} \alpha_i \, L_i Q^{-1} \tag{24}$$

guarantees both the asymptotical stability and $\|F_u(T,\Delta)\|_\infty < \gamma$, for all $\theta \in \Theta$ and $\Delta \in IQC(\psi, M)$.

Proof: From theorem 1, the robust and asymptotic stabilities for the closed-loop system (4) are satisfied if inequality (6) is guaranteed for all $\theta \in \Theta$ and $\Delta \in IQC(\psi, M)$. By multiplying both sides of (6) by $diag(P^{-1}, I, I)$ and its transpose respectively, and then defining $\lambda_1 = \gamma$ and $Q = P^{-1}$, this inequality will be as follows:

$$\begin{aligned} &\begin{bmatrix} He(A_{cl}(\theta)Q) & * & * \\ B_{1cl}^T(\theta) & 0 & * \\ B_{2cl}^T(\theta) & 0 & -\gamma^2 I \end{bmatrix} \\ &+ \begin{bmatrix} Q\,C_{2cl}^T(\theta) \\ D_{21cl}^T(\theta) \\ D_{22cl}^T(\theta) \end{bmatrix} (I) [C_{2cl}(\theta)Q \quad D_{21cl}(\theta) \quad D_{22cl}(\theta)] \\ &+ \lambda_2 \begin{bmatrix} QC_{1cl}^T(\theta) \\ D_{11cl}^T(\theta) \\ D_{12cl}^T(\theta) \end{bmatrix} M [C_{1cl}(\theta)Q \quad D_{11cl}(\theta) \quad D_{12cl}(\theta)] \\ &< 0 \end{aligned} \tag{25}$$

By using assumption 1 and schur complement of (25) related to the identity matrix I, equation (25) is satisfied if:

$$\begin{bmatrix} He(A_{cl}(\theta)Q) & * & * & * \\ B_{1cl}^T(\theta) & 0 & * & * \\ B_{2cl}^T(\theta) & 0 & -\gamma^2 I & * \\ C_{2cl}(\theta)Q & D_{21cl}(\theta) & D_{22cl}(\theta) & -I \end{bmatrix} + \Lambda < 0 \tag{26}$$

where

$$\begin{aligned} \Lambda = &\begin{bmatrix} C_{11cl}(\theta)Q & D_{111cl}(\theta) & D_{121cl}(\theta) & 0 \\ 0 & I & 0 & 0 \end{bmatrix}^T \begin{bmatrix} \Pi'_{11} & \Pi'_{12} \\ * & \Pi'_{22} \end{bmatrix} \\ &\times \begin{bmatrix} C_{11cl}(\theta)Q & D_{111cl}(\theta) & D_{121cl}(\theta) & 0 \\ 0 & I & 0 & 0 \end{bmatrix} \\ &\begin{bmatrix} \Pi'_{11} & \Pi'_{12} \\ * & \Pi'_{22} \end{bmatrix} = \lambda_2 \begin{bmatrix} \Pi_{11} & \Pi_{12} \\ * & \Pi_{22} \end{bmatrix} \end{aligned} \tag{27}$$

In (26), Λ can also be shown as:

$$\begin{aligned} \Lambda = &[C_{11cl}(\theta)Q \quad D_{111cl}(\theta) \quad D_{121cl}(\theta) \quad 0]^T \Pi'_{11} \\ &\times [C_{11cl}(\theta)Q \quad D_{111cl}(\theta) \quad D_{121cl}(\theta) \quad 0] \\ &+ \begin{bmatrix} 0 & * & * & * \\ (QC_{11cl}^T(\theta)\Pi'_{12}I)^T & He(D_{111cl}^T(\theta)\Pi'_{12}I) + I^T\Pi'_{22}I & * & * \\ 0 & D_{121cl}^T(\theta)\Pi'_{12}I & 0 & * \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{28}$$

Now, by replacing (28) in (26), inequality (26) is equivalent to:

$$\begin{bmatrix} He(A_{cl}(\theta)Q) & * & * \\ B_{1cl}^T(\theta) + (QC_{11cl}^T(\theta)\Pi'_{12}I)^T & He(D_{111cl}^T(\theta)\Pi'_{12}I) + I^T\Pi'_{22}I & * \\ B_{2cl}^T(\theta) & D_{121cl}^T(\theta)\Pi'_{12}I & -I \end{bmatrix} + \Lambda_{11} < 0 \tag{29}$$

where

$$\begin{aligned} \Lambda_{11} = &\begin{bmatrix} C_{11cl}(\theta)Q & D_{111cl}(\theta) & D_{121cl}(\theta) \\ C_{2cl}(\theta)Q & D_{21cl}(\theta) & D_{22cl}(\theta) \end{bmatrix}^T \begin{bmatrix} \Pi'_{11} & 0 \\ 0 & I \end{bmatrix} \\ &\times \begin{bmatrix} C_{11cl}(\theta)Q & D_{111cl}(\theta) & D_{121cl}(\theta) \\ C_{2cl}(\theta)Q & D_{21cl}(\theta) & D_{22cl}(\theta) \end{bmatrix} \end{aligned} \tag{30}$$

By schur complement of (29) related to $diag(\Pi'_{11}, I)$, this inequality is guaranteed if:

$$\begin{bmatrix} He(A_{cl}(\theta)Q) & * & * & * & * \\ B_{1cl}^T(\theta) + (QC_{11cl}^T(\theta)\Pi'_{12}I)^T & He(D_{111cl}^T(\theta)\Pi'_{12}I) + I^T\Pi'_{22} & * & * & * \\ B_{2cl}^T(\theta) & D_{121cl}^T(\theta)\Pi'_{12}I & -\gamma^2 I & * & * \\ C_{11cl}(\theta)Q & D_{111cl}(\theta) & D_{121cl}(\theta) & -{\Pi'_{11}}^{-1} & * \\ C_{2cl}(\theta)Q & D_{21cl}(\theta) & D_{22cl}(\theta) & 0 & -I \end{bmatrix} < 0 \tag{31}$$

On the other hand, the closed-loop system will be as follows where the controller is defined as (16).

$$\begin{aligned} A_{cl}(\theta) &= A(\theta) + B(\theta)K(\theta) \\ C_{11cl}(\theta) &= C_{11}(\theta) + D_{z1}(\theta)K(\theta) \\ C_{2cl}(\theta) &= C_2(\theta) + D_e(\theta)K(\theta) \\ B_{1cl}(\theta) &= B_1(\theta) \quad, B_{2cl}(\theta) = B_2(\theta) \\ D_{111cl}(\theta) &= D_{111}(\theta), D_{121cl}(\theta) = D_{121}(\theta) \\ D_{21cl}(\theta) &= D_{21}(\theta) \quad, D_{22cl}(\theta) = D_{22}(\theta) \end{aligned} \tag{32}$$

From (32) and (31), the asymptotical stability and $\|F_u(T,\Delta)\|_\infty < \gamma$ are guaranteed if the condition (33) is held for all $\theta \in \Theta$.

$$\begin{bmatrix} He(A(\theta)Q + B(\theta)K(\theta)Q) & * & * & * & * \\ B_1^T(\theta) + (\Pi'_{12}I)^T(C_{11}(\theta)Q + D_{z1}(\theta)K(\theta)Q) & He(D_{111}^T(\theta)\Pi'_{12}I) + I^T\Pi'_{22}I & * & * & * \\ B_2^T(\theta) & D_{121}^T(\theta)\Pi'_{12}I & -\gamma^2 I & * & * \\ C_{11}(\theta)Q + D_{z1}(\theta)K(\theta)Q & D_{111}(\theta) & D_{121}(\theta) & -{\Pi'_{11}}^{-1} & * \\ C_2(\theta)Q + D_e(\theta)K(\theta)Q & D_{21}(\theta) & D_{22}(\theta) & 0 & -I \end{bmatrix} < 0 \tag{33}$$

Equation (33) is a nonlinear inequality related to the matrices $K(\theta)$ and Q. For solving this problem we need to define the change variable.

$$L(\theta) = K(\theta)Q \tag{34}$$

By using (34), the inequality (33) is converted into a linear inequality by considering the matrices $K(\theta)$ and Q shown in equation (35).

$$\begin{bmatrix} He(A(\theta)Q + B(\theta)L(\theta)) & * & * & * & * \\ B_1^T(\theta) + (\Pi'_{12}I)^T\big(C_{11}(\theta)Q + D_{z1}(\theta)L(\theta)\big) & He(D_{111}^T(\theta)\Pi'_{12}I) + I^T\Pi'_{22}I & * & * & * \\ B_2^T(\theta) & D_{121}^T(\theta)\Pi'_{12}I & -\gamma^2 I & * & * \\ C_{11}(\theta)Q + D_{z1}(\theta)L(\theta) & D_{111}(\theta) & D_{121}(\theta) & -{\Pi'_{11}}^{-1} & * \\ C_2(\theta)Q + D_e(\theta)L(\theta) & D_{21}(\theta) & D_{22}(\theta) & 0 & -I \end{bmatrix} < 0 \tag{35}$$

From system matrices (15) and defining

$$L(\theta) = \sum_{i=1}^{r} \alpha_i L_i, \sum_{i=1}^{r} \alpha_i = 1, \alpha_i \geq 0 \tag{36}$$

i.e., $L(\theta)$ is polytopic, the inequality (35) can be represented as:

$$\sum_{i=1}^{r} \alpha_i^2 \varphi_{ii} + \sum_{i=1}^{r}\sum_{i<j} \alpha_i\alpha_j\left(\varphi_{ij} + \varphi_{ji}\right) = \sum_{i=1}^{r}\sum_{j=1}^{r} \alpha_i\alpha_j\varphi_{ij} < 0 \tag{37}$$

where φ_{ij} has been shown in(23). The LMIs (37) are guaranteed if (22) is feasible.■

Remark 2: The proposed technique in theorem 2 uses a simple procedure to convert the inequality (37) into a set of LMIs (22). It should be noted that this technique is also used in [25], [30].

Remark 3: In theorem 2, with respect to the scalar parameter λ_2 which is the degree of freedom, the inequalities will be nonlinear. Therefore, this parameter should be known before solving the LMIs (22). This parameter can be selected by optimization methods such as the Genetic algorithm (GA). Using this algorithm, for each fixed λ_2, the given LMIs in theorem 2 are solved and finally λ_2 is determined when the performance index γ is minimized.

Theorem 2 proposes an LMI condition to calculate a gain-scheduled controller where the time-domain IQC (ψ, M) known. If the objective is to design the gain-scheduled controller with respect to Δ, in order to maximize the stability margin, this theorem should be extended to solve the problem that given in theorem 3. In this case, the disturbance Δ with $\|\Delta\|_\infty < b$ is assumed for simplicity. The design objective is to calculate a gain-scheduled controller to achieve both the minimum robust gain γ and the maximum value of b, i.e., maximum stability margin.

Theorem 3: Consider the uncertain LPVS (14) where the IQC matrix ψ is defined as an identity system. If the matrices L_i,∀i $= 1,2,\dots,$r and $Q = Q^T > 0$, and minimum scalar parameters γ^2 and a_Δ are exist such that the following inequalities are feasible:

$$\begin{aligned} \Phi_{ii} < 0 \qquad & , i = 1,2,\dots,r \\ \Phi_{ij} + \Phi_{ji} < 0 \qquad & , i < j = 1,2,\dots,r \end{aligned} \tag{38}$$

where

$$\Phi_{ij} = \begin{bmatrix} He\big(A_iQ + B_iL_j\big) & * & * & * & * \\ B_{1i}^T & -I & * & * & * \\ B_{2i}^T & 0 & -\gamma^2 I & * & * \\ C_{11i}Q + D_{z1i}L_j & D_{111i} & D_{121i} & -a_\Delta I & * \\ C_{2i}Q + D_{ei}L_j & D_{21i} & D_{22i} & 0 & -I \end{bmatrix} \tag{39}$$

Then, the gain-scheduled controller (24) satisfies the asymptotical stability and $\|F_u(F_L(G,K),\Delta)\|_\infty < \gamma$ for all $\theta \in \Theta$ and $\|\Delta\|_\infty^2 < 1/a_\Delta$.

Proof: In this theorem, the disturbance Δ is considered as $\|\Delta\|_\infty < b$ whose the IQC matrix M will be as follows [5]:

$$M = diag(b^2 I, -I) \tag{40}$$

By considering M from (40) and equations (35)-(36) of theorem 2, the asymptotical stability and $\|F_u(F_L(G,K),\Delta)\|_\infty < \gamma$ for all $\theta \in \Theta$ and $\|\Delta\|_\infty < b$ are guaranteed if the inequality (41) is held.

$$\sum_{i=1}^{r} \alpha_i^2 \rho_{ii} + \sum_{i=1}^{r}\sum_{i<j} \alpha_i\alpha_j\left(\rho_{ij} + \rho_{ji}\right) = \sum_{i=1}^{r}\sum_{j=1}^{r} \alpha_i\alpha_j\,\rho_{ij} < 0 \tag{41}$$

where

$$\rho_{ij} = \begin{bmatrix} He\big(A_iQ + B_iL_j\big) & * & * & * & * \\ B_{1i}^T + (\Pi'_{12}I)^T\big(C_{11i}Q + D_{z1i}L_j\big) & -\lambda_2 I & * & * & * \\ B_{2i}^T & D_{121i}^T\Pi'_{12}I & -\gamma^2 I & * & * \\ C_{11i}Q + D_{z1i}L_j & D_{111i} & D_{121i} & -\frac{1}{b^2\lambda_2}I & * \\ C_{2i}Q + D_{ei}L_j & D_{21i} & D_{22i} & 0 & -I \end{bmatrix} \tag{42}$$

Inequality (41) is guaranteed if (38) is satisfied where λ_2 and b^2 have been selected 1 and $1/a_\Delta$ respectively. So the proof is done.■

Lemma 2: In theorems 2 and 3, if the system matrices B, D_z, and D_e are constant (not parameter dependent), the LMI conditions (22) and (38) are simplified to feasibility in every vertex of the parameters respectively or:

$$\varphi_i < 0 \qquad , i = 1,2,\dots,r \tag{43}$$

$$\Phi_i < 0 \qquad , i = 1,2,\dots,r \tag{44}$$

where φ_i and Φ_i will be as follows:

$$\varphi_i = \begin{bmatrix} He(A_iQ + BL_i) & * & * & * & * \\ B_{1i}^T + (\Pi'_{12}I)^T(C_{11i}Q + D_{z1}L_i) & He(D_{111i}^T\Pi'_{12}I) + I^T\Pi'_{22}I & * & * & * \\ B_{2i}^T & D_{121i}^T\Pi'_{12}I & -\gamma^2 I & * & * \\ C_{11i}Q + D_{z1}L_i & D_{111i} & D_{121i} & -{\Pi'_{11}}^{-1} & * \\ C_{2i}Q + D_eL_i & D_{21i} & D_{22i} & 0 & -I \end{bmatrix} \tag{45}$$

$$\Phi_i = \begin{bmatrix} He(A_iQ + BL_i) & * & * & * & * \\ B_{1i}^T & -I & * & * & * \\ B_{2i}^T & 0 & -\gamma^2 I & * & * \\ C_{11i}Q + D_{z1}L_i & D_{111i} & D_{121i} & -a_\Delta I & * \\ C_{2i}Q + D_eL_i & D_{21i} & D_{22i} & 0 & -I \end{bmatrix} \tag{46}$$

Proof: Let the matrices B, D_z, and D_e to be constant. Equation (37) in the proof of theorem 2 can be rewritten as follows:

$$\sum_{i=1}^{r} \alpha_i \varphi_i < 0 \tag{47}$$

where $\alpha_i \geq 0$. The inequality (47) is satisfied if $\varphi_i, \forall i = 1,..,r$ (in every vertex of the parameter box) is guaranteed. This issue can be represented in theorem 3 either. Therefore, the proof is completed.■

V. Numerical results

In this section, two examples are illustrated. In the first example, the aim is to derive an autopilot for the pursuit system to track the normal acceleration where the LPVS is considered as an uncertain LTI system and then the fixed controller is given by theorem 2. In the second one, a gain-scheduled controller will be obtained for the uncertain LPVS in order to satisfy the asymptotical stability and robust performance γ.

Example 1 (Autopilot design): In [31], first an linear parameter varying description of a pursuit system has been given at Mach 3 ($M = 3$) and the altitude 20000ft by:

$$\begin{bmatrix} \dot{\alpha}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} Z_\alpha & 1 \\ M_\alpha & 0 \end{bmatrix} \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} Z_\delta \\ M_\delta \end{bmatrix} \delta(t) \tag{48}$$
$$a_z(t) = N_a\, \alpha(t) + N_\delta\, \delta(t)$$

where $\alpha(t)$ is the angle of attack, $q(t)$ is the angular velocity, $\delta(t)$ is the deflection, $a_z(t)$ is the normal acceleration and the system parameters $Z_\alpha, Z_\delta, M_\alpha, M_\delta$, N_a and N_δ depend on the Mach number and the angle of attack changing over -15 to 15 degrees. Then, by defining

$$M_\alpha = M_{\alpha 0} + \theta\, M_{\alpha 1} = \frac{max(M_\alpha) + min(M_\alpha)}{2} + \theta \frac{max(M_\alpha) - min(M_\alpha)}{2}, |\theta| \leq 1 \tag{49}$$

where θ is time-varying and

$$M_\alpha = K_q M^2\big((2.15 \times 10^{-4})\alpha^2(t) - (1.95 \times 10^{-2})|\alpha(t)| + 5.1 \times 10^{-2}\big) \tag{50}$$

, it has been shown that the pursuit model (48) can be represented as Fig. 6 with the following state-space model:

$$\begin{bmatrix} \dot{\alpha}(t) \\ \dot{q}(t) \end{bmatrix} = \begin{bmatrix} a_Z M_{\alpha 0} + b_Z & 1 \\ M_{\alpha 0} & 0 \end{bmatrix} \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} a_Z \\ 1 \end{bmatrix} w(t) + \begin{bmatrix} Z_\delta \\ M_\delta \end{bmatrix} \delta(t) \tag{51}$$

$$z(t) = \begin{bmatrix} M_{\alpha 1} & 0 \end{bmatrix} \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix}$$
$$y(t) = \begin{bmatrix} 1 & 0 \\ a_N M_{\alpha 0} + b_N & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \alpha(t) \\ q(t) \end{bmatrix} + \begin{bmatrix} 0 \\ a_N \\ 1 \end{bmatrix} w(t) + \begin{bmatrix} 0 \\ N_\delta \\ 0 \end{bmatrix} \delta(t)$$

where

$$a_Z = 2.38 \times 10^{-3}, b_Z = -0.695 \qquad a_N = 4.59 \times 10^{-3}, b_N = -1.166 \tag{52}$$

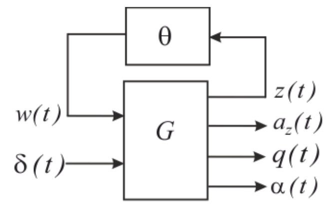

Fig. 6. LFT interconnection of the pursuit model

The weighted robust interconnection shown in Fig. 7 has been proposed to derive the H_∞ autopilot where the signals $e(t)$ and $d(t)$ in (14) will be as $[Z_e, Z_u]^T$ and $[A_{zc}, Noise, Dist]^T$, respectively.

Because $|\theta| \leq 1$, the IQC matrix M will be as follows [5]:

$$M = diag(I, -I) \tag{53}$$

Now, in theorem 2, by selecting:

$$\lambda_2 = 1.1387 \times 10^{-5} \tag{54}$$

, utilizing the Genetic algorithm and solving the inequality (22), the following autopilot guaranties l_2-gain $\gamma = 0.376$.

$$K = -[24.346 \quad 0.555 \quad 52.443] \tag{55}$$

The open-loop system (51) with the autopilot (55) has been evaluated by the step response shown in Fig. 8 when the time-varying parameter M_α will be changed as in Fig. 9. As suggested in [22], the autopilot can be also designed by the iterative-based methods in which the state-feedback controller can be obtained from [25]. To use this method, in the first step by assuming $\theta = 0$, the SOF controller is obtained as:

$$K = -[1.9 \quad 0.33 \quad 13.92] \tag{56}$$

that guarantees $\gamma = 0.13$ with known scalars $\rho = 0.0011$ and $\beta = 0.0853$. In the second step by considering $|\theta| \leq 1$, the controller (56) satisfies the stability and performance level $\gamma = 0.86$ by using the

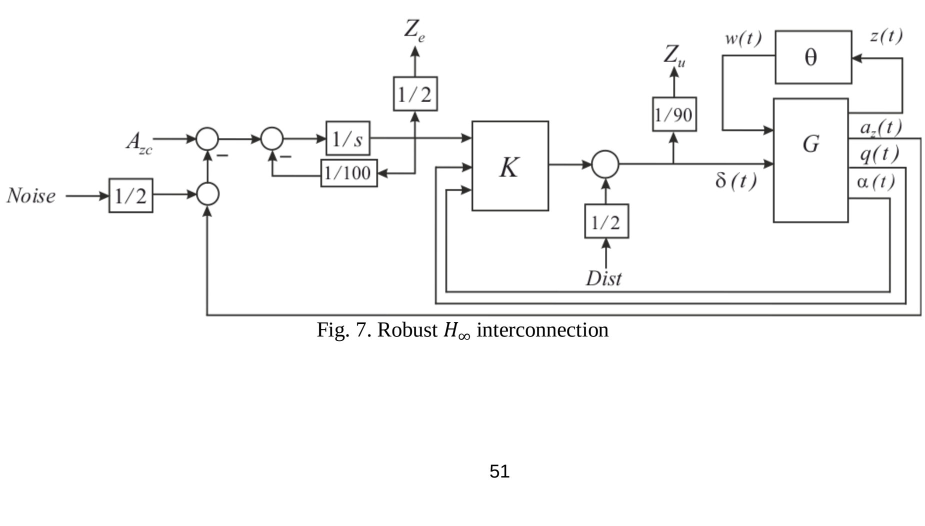

Fig. 7. Robust H_∞ interconnection

stability analysis presented in [22]. Consequently, the nonrepetitive method proposed in theorem 2 in which the controller and stability analysis are done simultaneously, can result in better performance (i.e., less γ) and less computational burden in comparison with the iterative method in [22]. Furthermore, simulation results illustrated in Fig. 8 imply that the closed-loop system appropriately tracks the acceleration step profile. Therefore, this example confirms the effectiveness of the proposed method in presence of the unmeasurable parameter M_α.

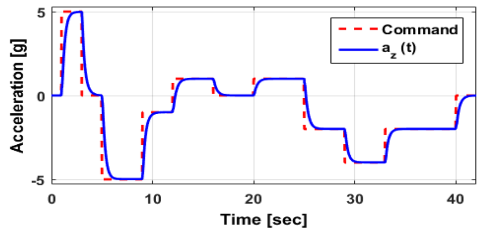

Fig. 8.The step response using the autopilot (55)

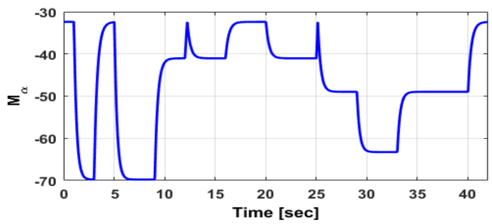

Fig. 9. Time-varying parameter M_α

Example 2 (An uncertain LPVS): Consider an uncertain model by the following state-space realization [32]:

$$\begin{gathered}\dot{x}(t)=\begin{bmatrix}\eta_1 & \eta_2\theta\\ -2 & -4\eta_2\end{bmatrix}x(t)+\begin{bmatrix}0\\ \eta_1\end{bmatrix}d(t)+\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}u(t)\\ e(t)=[1 \quad 0]x(t)\\ y(t)=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}x(t)\end{gathered} \tag{57}$$

where the time-varying parameter $\theta \in [2,4]$ is measurable, and the uncertainties η_1 and η_2 which are not available for the feedback interconnection, are between (0.9 to 1.1). So, the traditional methods for design of LPV controllers cannot be applied because all of the parameters are not measurable [27]. However, this problem can be solved by the IQC technique. For this purpose, by defining

$$\begin{gathered}\eta_1=1+\Delta_1, \eta_2=1+\Delta_2 \;, \Delta_1,\, \Delta_2\in[-0.1,0.1]\\ w(t)=\begin{bmatrix}\Delta_1 & \Delta_2\theta\\ 0 & -4\Delta_2\end{bmatrix}x(t)+\begin{bmatrix}0\\ \Delta_1\end{bmatrix}d(t)\\ =\begin{bmatrix}\Delta_1 & \Delta_2 & 0 & 0\\ 0 & 0 & \Delta_1 & \Delta_2\end{bmatrix}[z_1^T(t) \quad z_2^T(t) \quad z_3^T(t) \quad z_4^T(t)]^T\end{gathered} \tag{58}$$

The state-space matrices of the open-loop system (14) will be:

$$\begin{gathered}A=\begin{bmatrix}1 & \theta\\ -2 & -4\end{bmatrix}, B_1=B=C=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\\ C_{11}=\begin{bmatrix}1 & 0\\ 0 & \theta\\ 0 & 0\\ 0 & -4\end{bmatrix}, D_{121}=\begin{bmatrix}0\\ 0\\ 1\\ 0\end{bmatrix}, C_2=[0 \quad 1]\end{gathered} \tag{59}$$

with zero values for other matrices on (14). Furthermore, the IQC matrix M will be as (40) in which $b=0.2$, and the theorem 2 can be applied. By choosing $\lambda_2 = 30.48$ via the Genetic optimization, the following controllers will be obtained for two vertices that guaranty $\gamma = 2.61$.

$$\begin{gathered}K_1=-10^6\times\begin{bmatrix}1.5532 & 0.0006\\ 0.0001 & 5.2683\end{bmatrix}\\ K_2=-10^6\times\begin{bmatrix}1.3954 & 0.0001\\ 0 & 4.7064\end{bmatrix}\end{gathered} \tag{60}$$

Also, the scheduling parameters $\alpha_i, \forall i=1,2$ are chosen as:

$$\alpha_1=\frac{max(\theta)-\theta}{max(\theta)-min(\theta)}, \alpha_2=\frac{\theta-min(\theta)}{max(\theta)-min(\theta)} \tag{61}$$

Therefore, the LPV controller (60) should be interpolated by the scheduling parameters (61) in the real-time, when the fixed-controllers are calculated as off-line. This example shows that the proposed method in theorem 2 can be applied to control of the LPVSs where all the parameters are unavailable or cannot be estimated for the feedback interconnection. Now, if the goal is to define the maximum stability margin of Δ_1 and Δ_2, theorem 3 can be applied, and by solving inequalities (38), the performance index $\gamma = 3.49$ and $b = 3.49$ (i.e., $\alpha_\Delta = 0.0818$ or $\|\Delta\|_\infty^2 < 12.225$) will be obtained. For instance, the absolute of the uncertainties Δ_1 and Δ_2 can change to less than 1.75. Also, the controller can be obtained by the state-space feedback method introduced in [32] in which system (57) is considered as a polytopic model with the uncertainty over its vertices. By using the suggested method in [32], eight inequalities should be solved. Nevertheless, the proposed method in theorem 2 needs to solve only three inequalities which results in a less computational burden.

VI.CONCLUSION

In this paper, the LMI-based algorithm has been given to synthesizing a robust gain-scheduled state-feedback controller for a class of uncertain LPVS. Also, the LPV design step and the IQC analysis step have been considered simultaneously to achieve a better performance and less computational burden. The proposed method guarantees the closed-loop asymptotical stability when the induced l_2-gain γ minimizes for all $\theta \in \Theta$. Finally, the proposed method has been evaluated on two examples. In the first example, it has been shown that this method can be applied to the LTI system with a block uncertainty in form of the upper LFT representation. The effectiveness of the proposed method against the unmeasurable parameters in the uncertain LPVS has been shown in the second example. In these two examples, less conservatism and less computational burden of the proposed method have been confirmed in comparison with state-of-the-art methods.

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